Zubair Uniform Distribution with SQC Application

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Abstract

The main components of the statistical process control methodology are frequency distributions of process and quality characteristic of the data and control charts are some manufacturing processes have skewed distributions by nature. In this situation, conventional control charts based on the supposition of normality can result in incorrect inferences about the stability and capabilities of the process. Such erroneous inferences would increase the cost of production and result in client loss to rivals. Many researchers have introduced control charts based on various non – normal distributions according to the process type they are dealing with in order to avoid such issues. In this study, the performance of the production process is assessed using control charts created by using the Zubair uniform Distribution.

Keywords*: Statistical Quality control, Control charts, Zubair Uniform distribution*

1. Introduction

A frequently utilised methodology to enhance processes in contemporary production and service businesses is statistical process control. This methodology primarily relies on frequency distributions of process and quality attribute data and control charts. The Shewhart control chart, created by professor Shewhart Walter, was popular and well-known in the 1920s. Since then, other researchers have created various control charts. Customers actually demand that businesses provide the best goods and services, together with panels that aid in ongoing process improvement, making businesses more dependable and capable of producing goods of a high calibre, satisfying client requirements, and achieving societal greatness. When random causes of variability are present in the system, Shewart control charts are created under the premise that the process being monitored produces a quality characteristic that may be approximated by a symmetrical normal distribution. If the samples being measured and watched are large enough, the central limit theorem can be used to approximate distributions to the normal distribution. This cannot be guaranteed in many industrial situations, and the process output may not always be evenly distributed, heavy tailed,

and skewed. The distributions in several production processes are frequently skewed. In these circumstances, conventional control charts based on the supposition of normality can result in incorrect inferences about the competence and stability of the process. Such erroneous inferences would increase the cost of production and result in client loss to rivals. This can be overcome by comprehending distributions that serve as good models for the kind of quality features. The paper is organized in the manner described below. In section 2, the pertinent literature is evaluated. Section 3 provides a description of the Zubair uniform distribution and its measurements. Section 4 provides control limits identifying out of control for the real life data. Summary is provided in section 5.

2. Review of literature

A frequently utilized methodology to enhance processes in contemporary production and service businesses is statistical process control. This methodology primarily relies on frequency distributions of process and quality attribute data and control charts. Amin and Venkatesan (2017 & 2019) suggested new developed in control chart strategies and made comparison with Bayesian method of control chart. Further, Amin and Venkatesan (2017 & 2019) have studied control chart for non- normal cases. To convert non-normal data into calculations for process control and process capability, Derya and Canan (2012) created standard control charts based on the weibull, lognormal, and exponential distributions. The control charts were most recently proposed by Santiago and Smith (2013). To approximate normal data from exponentially distributed data, they employed Nelson's (1994) variable transformation. the usage of charts that can be used for process optimization using this distribution as a model. When it was established that the customary practise of using standard distributions was inappropriate, Rao (1965) developed it and expressed it generally in connection to the modelling of statistical data. It was not proper to use standard distributions when Zubair uniform distributions were being used to modify practise. The model specification and date interpretation issues can be collectively accessed using the Zubair uniform distribution. By altering the probabilities of events occurring in reality to arrive at a specification of the probabilities of those events as observed and recorded, the Zubair uniform distributions take into consideration the technique of ascertainment. To track the process, various distributions have been created using the Zubair uniform distribution.

3. Description of the distribution

Definition: A random variable X has a Zubair Uniform distribution if the density function is given by

$$f(x) = \frac{2\alpha \frac{1}{(b-a)}\left(\frac{x-a}{b-a}\right) e^{\left\{\alpha\left(\frac{x-a}{b-a}\right)^2\right\}}}{\left(e^{\alpha}-1\right)} \tag{1}$$

Where **'a'** and **'b'** are the shape and scale parameters of the distribution respectively. The moment generating function can be obtained by

$$M_X(t) = \int_{-\infty}^{\infty} e^{tx} f(x) dx$$

$$= \frac{2\alpha}{(b-a)^2\left(e^{\alpha}-1\right)} \int_a^b e^{tx}(x-a)\left(e^{tx+\alpha\left(\frac{x-a}{b-a}\right)^2}\right) dx$$

One gets after simplifications

$$M_X(t) = \frac{e^{bt+\alpha(b-a)^2}}{(b-a)^2\left(e^{\alpha}-1\right)}\left[\frac{1}{b-a} - \frac{1}{2\alpha + t(b-a)}\right] \tag{2}$$

In order to use this distribution, one need study the mean and variance of the distribution. The mean and variance are obtained and are respectively are given by

$$E(X) = \frac{e^{\alpha(b-a)^2}}{(b-a)^3\left(e^{\alpha}-1\right)}\left(b - \frac{b}{\left(t + \frac{2\alpha}{b-a}\right)} + \frac{1}{\left(t + \frac{2\alpha}{b-a}\right)^2}\right) \tag{3}$$

$$V(X) = \frac{e^{\alpha(b-a)^2}}{(b-a)^3\left(e^{\alpha}-1\right)}\left(\begin{array}{l} 1 + \frac{e^{\alpha(b-a)^2}}{(b-a)^3\left(e^{\alpha}-1\right)}\left(b - \frac{b(b-a)}{2\alpha}\right) + \frac{2(b-a)^2}{4\alpha^2}\left(b - \frac{b(b-a)}{2\alpha}\right) \\ + \left(1 + b + \frac{e^{\alpha(b-a)^2}}{(b-a)^3\left(e^{\alpha}-1\right)}\frac{(b-a)^2}{4}\right)\left(\frac{(b-a)^2}{4\alpha^2} - \frac{2(b-a)}{2\alpha}\right) \end{array}\right) \tag{4}$$

4. Control limits

Three sigma UCL and LCL are obtained as specified by Montgomery (2012) and one can get the control limits for Zubair Uniform distribution using (3) and (4) and are given by

$$UCL = \overline{X} + 3A$$

$$CL = \overline{X}$$

$LCL = \overline{X} - 3A$, When A is a function of **'a', 'b',** α and given by

$$A = \frac{e^{\alpha}(b-a)^{2}}{(b-a)^{3}(e^{\alpha}-1)}\left\{\begin{array}{l} 1 + \dfrac{e^{\alpha}(b-a)^{2}}{(b-a)^{3}(e^{\alpha}-1)}\left(b - \dfrac{b(b-a)}{2\alpha}\right) \\ + \dfrac{2(b-a)^{2}}{4\alpha^{2}}\left(b - \dfrac{b(b-a)}{2\alpha}\right) \\ + \left(1 + b + \dfrac{e^{\alpha}(b-a)^{2}}{(b-a)^{3}(e^{\alpha}-1)}\dfrac{(b-a)^{2}}{4}\right) \\ \left(\dfrac{(b-a)^{2}}{4\alpha^{2}} - \dfrac{2(b-a)}{2\alpha}\right) \end{array}\right\}$$

$$UCL = \frac{e^{\alpha}(b-a)^{2}}{(b-a)^{3}(e^{\alpha}-1)}\left[b - \frac{b}{\left(t + \frac{2\alpha}{b-a}\right)} + \frac{1}{\left(t + \frac{2\alpha}{b-a}\right)^{2}}\right] + 3\sqrt{A}$$

$$CL = \frac{e^{\alpha}(b-a)^{2}}{(b-a)^{3}(e^{\alpha}-1)}\left[b - \frac{b}{\left(t + \frac{2\alpha}{b-a}\right)} + \frac{1}{\left(t + \frac{2\alpha}{b-a}\right)^{2}}\right]$$

$$LCL = \frac{e^{\alpha}(b-a)^{2}}{(b-a)^{3}(e^{\alpha}-1)}\left[b - \frac{b}{\left(t + \frac{2\alpha}{b-a}\right)} + \frac{1}{\left(t + \frac{2\alpha}{b-a}\right)^{2}}\right] - 3\sqrt{A}$$

5. Numerical Illustration

An example regarding the construction of control limits is considered for illustrating the applications of the proposed method. The control limits of the Zubair Uniform distribution are obtained using simulated data set for parameters **'a', 'b'** and α. All the generated samples are reported in Table 1.

Data Set:

A production manager for a tire company has inspected the number of defective tires in five random samples with 20 tires in each sample. The table below shows the number of defective tires in each sample of 20 tires.

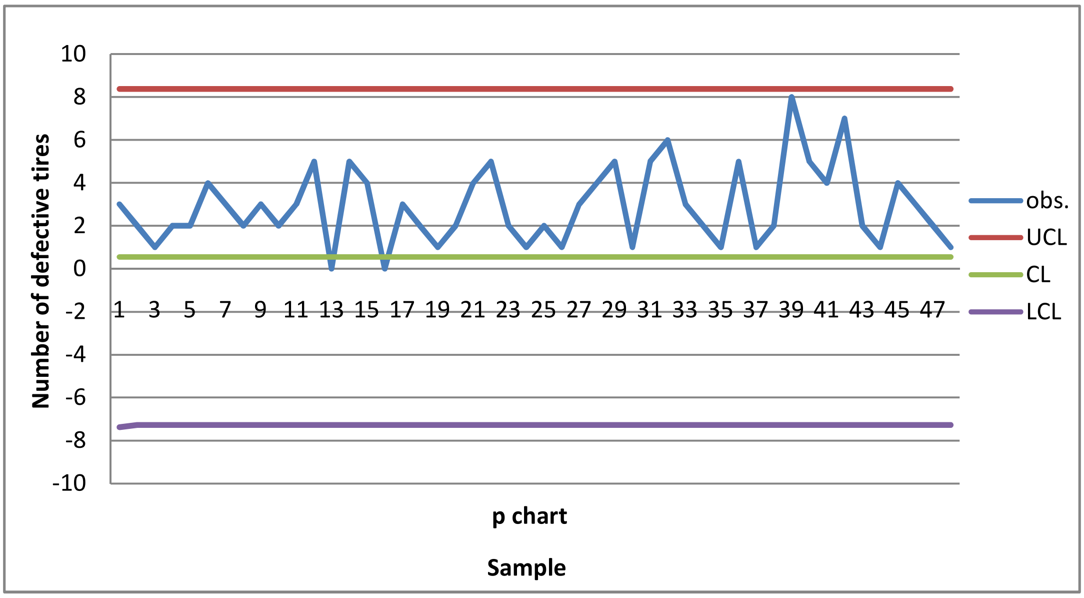
3,	2,	1,	2,	2,	4,	3,	2,	3,	2,	3,	5,
0,	5,	4,	0,	3,	2,	1,	2,	4,	5,	2,	1,
2,	1,	3,	4,	5,	1,	5,	6,	3,	2,	1,	5,
1,	2,	8,	5,	4,	7,	2,	1,	4,	3,	2,	1.

Control limits for the above data have been computed using different values for the parameter 'a', 'b' and are summarized in table 1.

Table 1: Control limits Using Zubair Uniform Distribution

b	a	α=1			α=3		
		UCL	**CL**	**LCL**	**UCL**	**CL**	**LCL**
1	-0.3	51.34	12.13	-27.08	6.34	0.70	-4.94
	-0.5	28.83	7.41	-14.01	4.93	0.61	-3.71
	-0.7	73.46	3.98	-65.50	5.08	0.58	-3.92
	-0.9	31.81	2.98	-25.85	8.38	0.55	-7.28
3	-0.3	38.27	1.26	-35.85	30.21	19.98	9.75
	-0.5	18.53	0.35	-6.04	38.14	26.14	14.14
	-0.7	12.48	1.35	-9.78	14.17	3.47	-15.10
	-0.9	11.43	2.67	-6.09	21.38	0.94	-23.26

It is observed from the Table1, that for the fixed value of the parameter '**b**', with '**a**' as random variable that target value α increase whenever the parameter b increases and the deviations limits also increase, the Zubair uniform control chart is given in fig.1 for **a= - 0.9, b= 1 and** $\alpha = 3$



It is also observed that, the observations of the process control must be 0<X<a; b>0; $\alpha > 0$, otherwise the process will be out of control. That is, depends of the manufacturing products, the manufacturing engineers should fix the parameters value based on what type of data they are working with. As it can be seen on Table 1. The increasing of parameter '**b**', the parameter a should be low in order to get the control limits because whenever the parameter '**b**' increase with lesser parameter '**a**', the variance values become negative also if b tends to 0.

6. Conclusions

In this paper, a Zuabir Uniform distribution can be used for process quality control. When the study variable is non-normal distribution. When the parameters a, b, and have different values, the control limits for the zubair uniform distribution are also derived and illustrated with standard data and it is found to be more suitable for the data when non-normality is assured.

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