Five Dimensional FRW Cosmological Model in the Presence of a Perfect Fluid Corresponding to Various Models in Brans-Dicke Theory of Gravitation

Jagat Daimary, Rajshekhar Roy Baruah (Corresponding Author)
Department of Mathematical Sciences, Bodoland University, Kokrajhar

Abstract:

Five dimensional FRW space-time is considered in the presence of perfect fluid proposed by Brans-Dicke [1] theory of gravitation. Stiff fluid, disordered radiation, dust and false vacuum models are considered in the framework of Brans-Dicke theory of gravitation. Physical properties and kinematical properties of the models are also discussed.

Keywords: Perfect Fluid Models; Stiff Fluid Model; Radiating Model; Dust Model; False Vacuum Model; Brans-Dicke Theory; FRW Models.

I. INTRODUCTION

Perfect fluid is the most frequently encounter fluid in modelling our universe. Perfect fluid physicists is those fluid which can be completely characterised by their rest frame pressure and density. In perfect fluid, pressure is isotropic, so in all direction the pressure of perfect fluid is same. There is no shear stress, no conduction, and no viscosity in a perfect fluid. We know that the matter in the universe obeys the perfect fluid hypothesis. So the large number of investigations in relativistic cosmology pertains to situations of perfect fluid distributions. The limiting case of equation of state p=ρ is known as stiff fluid or Zel'dovich fluid is considered by various authors for constructing models of the universe. The solution of the Einstein's field equations with the "stiff equation" of state are considered in several contexts, mostly of an astrophysical nature, particularly in situations involving "white holes" lagging cores of a big-bang cosmology, rotating neutron stars (pulsars) or a central region of quasars and inhomogeneous cosmological models. The equation of state defined by a special barotropic fluid is given by p+ρ=0. The condition p+ρ=0, ρ>0 satisfying by the fluid is called ρ vacuum" or "degenerate vacuum" or "false vacuum" [Davies, [2]; Blome and Priester, [3]; Hogan [4] and Kaiser and Stebbins, [5]]. The fluid for the equation of state p=0, ρ≠0 is known as dust distribution and the fluid for ρ=4p is known as disorder distribution (Radiating model) respectively. During condensed phase of the evolution the concept of dust as well as the fluid breaks down (Bichteler, [6]). In this case quantum mechanical description has to be used. For p (=ρ) = 0 (i.e. when pressure and energy density are zero), then this case is known as vacuum case. In Case of vacuum the space time does not contain any matter i.e. $T_{\mu\upsilon} = 0$.

By discussed the various thoughts around Mach's principle the Brans and Dicke being an outcome. The idea of Mach that the phenomenon of inertia arises from accelerations with respect to the general mass distribution of the universe is the starting point of Brans and

Dicke. So the inertial masses of the various elementary particles ought not to be fundamental constant but should rather represent the particles interaction with some cosmic field. In particular Brans and Dicke theory of gravity is the most important one among all other existing alternative theories of gravity. The Saez and Ballester [7] are playing very important role in the discussion of modern cosmology. The recent inflationary model (Mthiazhagan and Johri [8]), extended inflationary model (La and Steinhardt [9]), hyper extended inflationary model (Steinhardt and Aceeta [10]), Chaotic inflation (linde [11]) are based on Brans and Dicke theory of gravitation. Brans and Dicke defined a scalar-tensor theory of gravitation involving a scalar field ϕ in addition to the familiar general relativistic metric tensor field g_{ij} . In Brans and Dicke theory, instead of a gravitational field, a dynamical scalar field has been introduced to account for the variable gravitational constant to incorporate Mach's Principle. The Brans and Dicke field equations for combined scalar and tensor fields are define by,

$$R_{ij} - \frac{1}{2} g_{ij} R + \frac{\omega}{\phi^2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2} g_{ij}\phi_{,k}\phi^{,k}\right) + \frac{1}{\phi}[\phi_{i;j} - g_{ij}\Box\phi] = 8\pi\phi^{-1}T_{ij} \quad (1)$$

$$\Box\phi = \phi_{;i}^{,i} = \frac{8\pi T}{3+2\omega} \quad (2)$$

which is a consequence of the field equations (1) and (2), where ϕ is the scalar field, ω is the dimensionless coupling constant and T_{ij} is the stress energy tensor of the matter. Here comma and semicolon represent partial and covariant differentiation respectively.

The Friedman-Robertson-Walker (FRW) model defined spatially homogeneous and isotropic universe in four dimensions. The study of five dimensional space-time has been an immense interest in recent years because of the fact that the cosmos at its early stage of evolution of the universe might have had a higher dimensional era. Marciano [12] has suggested that the experimental detection of the fundamental constants with varying time could produce the evidence for the existence of extra dimensions. Many rechargers to the field equation of a higher dimensions Witten [13], Appelquist et al. [14], Chodos and Detweiler [15] were attracted to the study of higher dimensional cosmology because the universe has undergone compactification transitions.

Many authors investigated five dimensional cosmological models in Brans and Dicke theory of gravitation. (Reddy et al [16]; Reddy et al. [17]; Amir F. Bahrehbakhsh et al. [18]; Singh and Singh [19]; Naidu et al. [20], Reddy and Vijayalakshmi [21], V.U.M. Rao et al. [22]; T.Ramprasad et al. [23]; D.R.K. Reddy et al. [24]; G. P. Singh and Binaya K. Bishi [25]; V.U.M.Rao et al. [26]; Y. Aditya and D. R. K. Reddy [27] and Jianbo Lu1 et al. [28]). Recently, Rami Ahmad El-Nabulsi [29] have discussed about Five-dimensional Brans Dicke compactified universe dominated by a varying speed of light.

Motivated by the above study and discussion, in this paper we investigated five dimensional FRW Zeldovich or Stiff fluid, Radiating, Dust and False vacuum or inflationary models in Brans and Dicke theory of gravitation.

II. METRIC AND FIELD EQUATIONS

Let us consider five dimensional FRW space time metric in the following form

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{(1-kr^2)} + r^2(d\theta^2 + \sin^2\theta d\,\phi^2) + (1-kr^2)d\psi^2\right] \tag{3}$$

Where a(t) is the expansion scale factor of the universe and k=1, 0, -1, for space of positive, vanishing and negative curvature parameter representing closed, flat and open models of the universe respectively.

For the metric (3), the non-zero components of the Einstein tensor are

$$G_1^1 = G_2^2 = G_3^3 = G_4^4 = 3\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2} + 3\frac{k}{a^2} \tag{4}$$

$$G_5^5 = 6\frac{\dot{a}^2}{a^2} + 6\frac{k}{a^2} \tag{5}$$

Here an overhead dot represents differentiation with respect to cosmic time t.

The energy momentum tensor in the presence of a perfect fluid is given by

$$T_j^i = (\rho + p)u^i u_j - g_{ij}p \qquad \text{i, j = 1,2,3,4,5} \tag{6}$$

Here, $u^i u_i = 1,\; u^i u_j = 0$
(7)

$$T_{;j}^{ij} = 0 \tag{8}$$

Therefore,

$$T_5^5 = \rho; \quad T_1^1 = T_2^2 = T_3^3 = T_4^4 = -p \tag{9}$$

$$T = T_1^1 + T_2^2 + T_3^3 + T_4^4 + T_5^5 = \rho - 4p \tag{10}$$

III. Solutions and the models

Now applying co moving coordinates, the field equations (1)-(2) with the help of equations (4)-(10), for the FRW metric space (3), can be expressed as

$$6\frac{\dot{a}^2}{a^2} + 6\frac{k}{a^2} + \frac{\omega}{2}\frac{(\dot{\phi})^2}{\phi^2} + 4\frac{\dot{a}\dot{\phi}}{a\phi} = 8\pi\phi^{-1}\rho \tag{11}$$

$$3\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2} + 3\frac{k}{a^2} - \frac{\omega}{2}\frac{(\dot{\phi})^2}{\phi^2} + 3\frac{\dot{a}\dot{\phi}}{a\phi} + \frac{\ddot{\phi}}{\phi} = 8\pi\phi^{-1}p \quad (12)$$

$$\ddot{\phi} + 4\frac{\dot{a}\dot{\phi}}{a} = 8\pi(3+2\omega)^{-1}(\rho - 4p) \quad (13)$$

$$\dot{\rho} + 4\frac{\dot{a}}{a}(\rho + p) = 0 \quad (14)$$

The Physical parameters of the observational interest are, Hubble parameter H and deceleration parameter q which are given by

$$H = \frac{\dot{a}}{a} \quad (15)$$

$$q = \frac{-(\dot{H} + H^2)}{H^2} \quad (16)$$

Field equation (11)-(14) are three independent equations in four unknowns a, ϕ, p, and ρ [equation (14) being the consequence of equation (11)-(13)]. So we need an extra condition to obtain a determinate solution. Here we find five dimensional FRW flat models (i.e, k=0). Let us consider the well accepted relation between scalar field ϕ and the scale factor of the universe a(t) defined by

$$\phi = \phi_0 a^n \quad (17)$$

where ϕ_0 and n > 0 are constants. Here we define the solution of the field equations for the following physically important models in this particular case.

3.1. Case (1): Zeldovich or Stiff fluid model (i.e, ρ=p)

In this case using equation (17), the field equations (11)-(13) gives the following solutions for the scale factor

$$a(t) = [(n+4)(a_0 t - t_0)]^{\frac{1}{n+4}} \quad (18)$$

where the scalar field is given by

$$\phi = \phi_0[(n+4)(a_0 t - t_0)]^{\frac{n}{n+4}} \quad (19)$$

Now with the proper choice of coordinates and integration constant the five dimensional FRW flat models, in this particular case becomes (i.e. $a_0 = 1$)

$$ds^2 = dt^2 - [(n+4)(t-t_0)]^{\frac{2}{n+4}}[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) + d\psi^2] \quad (20)$$

Along with the scalar field, in the model, given by

$$\phi = \phi_0[(n+4)(t-t_0)]^{\frac{n}{n+4}} \quad (21)$$

Equation (20) defined five dimensional flat FRW cosmological model corresponding to stiff fluid in Brans-Dicke scalar-tensor theory of gravitation with the following physical and kinematical parameters which are important in the discussion of cosmology.

The spatial volume in this case is given by

$$V = a^4 = [(n+4)(t-t_0)]^{\frac{4}{n+4}} \tag{22}$$

The Hubble parameter H is defined by,

$$H = \frac{1}{(n+4)(t-t_0)} \tag{23}$$

Deceleration parameter q is

$$q = n+3 \tag{24}$$

In this case, ρ = p, so the energy density and pressure are given by

$$8\pi\rho = 8\pi p = \phi_0 \left[\frac{1}{(n+4)(t-t_0)}\right]^{\frac{n+8}{n+4}} [\frac{\omega n^2}{2} + 4n + 6] \tag{25}$$

From equation (22), (23), (24) and (25) we obtained that the volume scale factor of the universe increases with the growth of cosmic time which shows the spatial expansion of the universe. At t=0 (i.e, at the initial epoch), the Hubble parameter, the energy density and pressure assume infinitely large values whereas with the growth of cosmic time they decrease to null values at t→∞. We have also found that the deceleration parameter of the model is q > 0. We know that if q < 0 the model accelerates and when q > 0, the model decelerates in the standard way. Hence in this case the model in five dimensions decelerates in the standard way which is not in accordance with the present day scenario of accelerated expansion of the universe. However, the universe will accelerate in finite time after compactification transition and cosmic re-collapse where the universe in turn inflates, decelerates and then accelerates (Nojri and Odinstov [30]).

3.2. Case (ii): Radiating models in five dimensions

In this case ρ=4p, hence the field equations (11)-(13) for flat models (i.e, k=0) reduces to two independent equations

$$6\frac{\dot{a}^2}{a^2} + \frac{\omega}{2}(\frac{\dot{\phi}}{\phi})^2 + 4\frac{\dot{a}\dot{\phi}}{a\phi} = 32\pi\phi^{-1}p \tag{26}$$

$$3\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2} - \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + 3\frac{\dot{a}\dot{\phi}}{a\phi} + \frac{\ddot{\phi}}{\phi} = -8\pi\phi^{-1}p \tag{27}$$

To find a determinate solution of the field equations (26) and (27), we apply the special law of variation for the Hubble's parameter proposed by Berman which is define by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = constant \tag{28}$$

Here q is the constant deceleration parameter of the model of the universe.

From equation (28), we get

$$a(t) = (ct + d)^{\frac{1}{1+q}} \tag{29}$$

where $c \neq 0$ and d are constant of integration. From equation (29) we get $1+q > 0$. Now using equation (29) the field equations (26) and (27) after redefining the constants gives the metric coefficients as

$$a(t) = (t - t_0)^{\frac{1}{1+q}} \tag{30}$$

where the scalar field is given by

$$\phi = \phi_0 (t - t_0)^{\frac{n}{1+q}} \tag{31}$$

Now, for flat model (i.e. k=0) in this case the metric (3) is defined by

$$ds^2 = dt^2 - (t - t_0)^{\frac{2}{1+q}} [dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2) + d\psi^2] \tag{32}$$

The model (32) along with equation (31) represent a flat FRW five dimensional cosmological model in Brans-Dicke scalar-tensor theory of gravitation which can be considered as analogous to radiating model of general relativity in four dimensions. This universe describes the early stages of evolution of the universe with the following physical and kinematical parameters of the model which are important in the physical discussion of cosmology.

In this case spatial volume V is define by

$$V = a^4 = (t - t_0)^{\frac{4}{1+q}} \tag{33}$$

The Hubble's parameter H is given by

$$H = \frac{\dot{a}}{a} = \frac{1}{(1+q)(t - t_0)} \tag{34}$$

Energy density is

$$8\pi\rho = \phi_0 \frac{(t-t_0)^{\frac{n}{1+q}}}{[(1+q)(t-t_0)]^2}[\frac{\omega n^2}{2} + 4n + 6] \qquad (35)$$

The pressure is define by

$$32\pi p = \phi_0 \frac{(t-t_0)^{\frac{n}{1+q}}}{[(1+q)(t-t_0)]^2}[\frac{\omega n^2}{2} + 4n + 6] \qquad (36)$$

From the above result we obtain that the universe in this case also behaves in the similar way as in the case (i) of stiff fluid universe since 1+q > 0.

3.3. Case (iii): dust model (i.e. p=0)

In this case for flat model (i.e. k=0), equations (11)-(13) gives the two independent equations

$$6\frac{\dot{a}^2}{a^2} + \frac{\omega}{2}(\frac{\dot{\phi}}{\phi})^2 + 4\frac{\dot{a}\dot{\phi}}{a\phi} = 8\pi\phi^{-1}\rho \qquad (37)$$

$$3\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2} - \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + 3\frac{\dot{a}\dot{\phi}}{a\phi} + \frac{\ddot{\phi}}{\phi} = 0 \qquad (38)$$

Proceeding in the similar way as in the case (ii) we obtain the same model given by equation (32) with the scalar field, the energy density, and the pressure given by equation (31), (35) and (36) respectively. The physical behavior of the universe is the same as in the case (ii) except that p=0.

3.4. Case (iv): false vacuum or inflationary model (i.e. ρ+p = 0)

In this case for flat model (i.e. k=0) the field equations (11)-(13) given by

$$3\frac{\dot{a}^2}{a^2} + \omega(\frac{\dot{\phi}}{\phi})^2 + \frac{\dot{a}\dot{\phi}}{a\phi} - 3\frac{\ddot{a}}{a} - \frac{\ddot{\phi}}{\phi} = 0 \qquad (39)$$

Proceeding in the similar way as in the case (ii) we get the same model given by equation (32) along with the scalar field given by equation (31) satisfying the following relation [using equation (30) and (31) in equation (39)]

$$\frac{[n^2(\omega-1) + n(q+2) + 3(1+q)]}{[(1+q)(t-t_0)]^2} = 0 \qquad (40)$$

In this model the pressure is given by

$$8\pi p = -\phi_0 \frac{(t-t_0)^{\frac{n}{1+q}}}{[(1+q)(t-t_0)]^2}\left[\frac{\omega n^2}{2} + 4n + 6\right] \tag{41}$$

From equation (41) it is observed that pressure p is negative which shows that the universe is in an accelerating mode which is in accordance with the recent scenario of the accelerated expansion of the universe with exotic pressure.

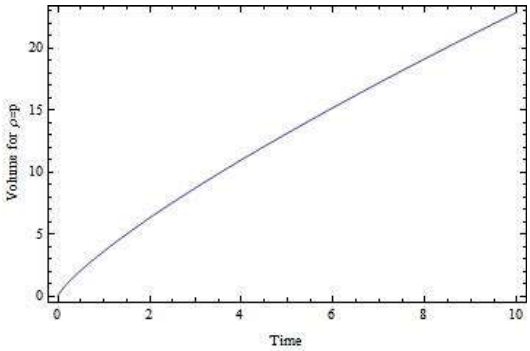

Fig-1. Volume vs. time
(π=3.14, n=1; $t_0 = 0$; ω=500, $\phi_0 = 0.001$)

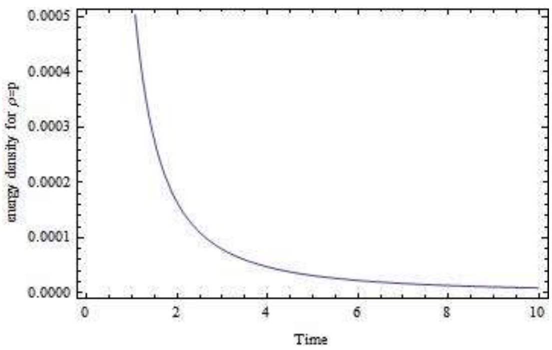

Fig-2. Density vs. time for k=0
(π=3.14, n=1, $t_0 = 0$; ω=500, $\phi_0 = 0.001$)

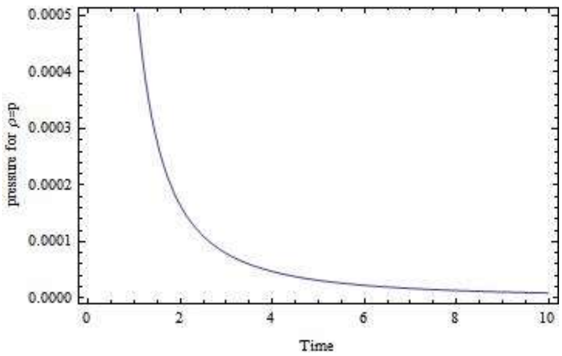

Fig-3. Pressure vs. time for k=0
(π=3.14, n=1, ω=500, $\phi_0 = .001$)

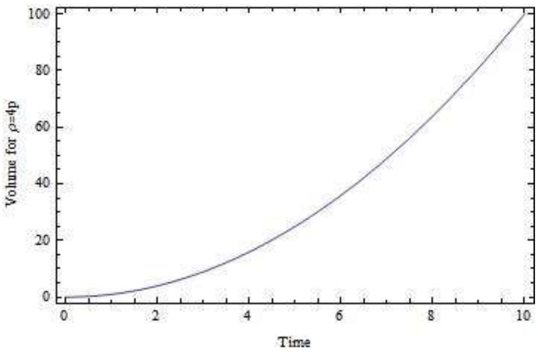

Fig-4. Volume vs. time
(π=3.14, n=1, ω=500, $\phi_0 = .001$)

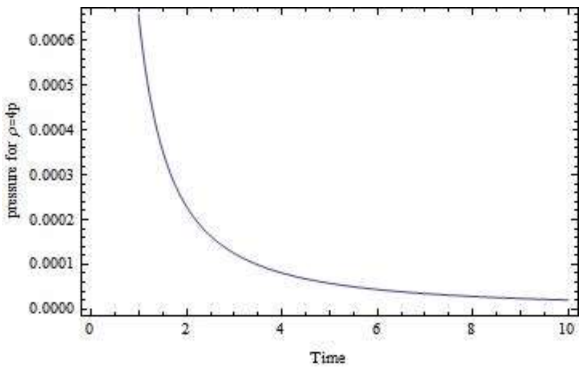

Fig-5. Density vs. time for k= 0
(п=3.14, q=1, n=1, ω=500, $\phi_0 = 0.001$)

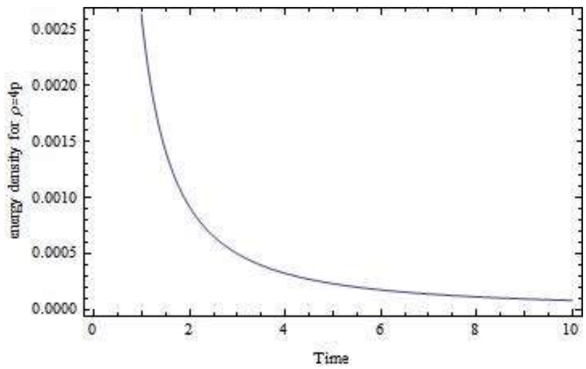

Fig-6. Pressure vs. time for k= 0
(п=3.14, q=1, n=1, ω=500, $\phi_0 = 0.001$)

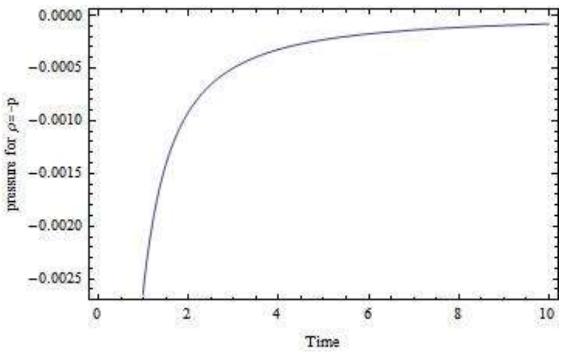

Fig-7, Pressure vs. time for k= 0
(п=3.14, q=1; n=1, ω=500, $\phi_0 = 0.001$)

IV. PHYSICAL INTERPRETATION

Equation (32) represents a five dimensional FRW flat model for radiating, dust and false vacuum in the scalar-tensor theory of gravitation proposed by Brans and Dicke. For Case (i) model the spatial volume is given by equation (22) and for Case (ii), case (iii) and case (iv) are given by equation (33) which shows spatial expansion of the models with the increase in cosmic time t. The average Hubble parameter in case (i) is given by equation (23) and for case (ii), case (iii), case (iv) are given by equation (34) which diverges at t = t_0 and will vanish for infinitely large t. Except case (iv) it is obtained that the pressure and energy density in all the models diverge at t = t_0 and vanish as t→∞. From equation (41) we find that in the case (iv) (i.e., false vacuum or inflationary model) the pressure becomes negative which shows that the universe, in this particular case, is in an accelerating mode. This is in accordance with the recent scenario of early inflation and late time acceleration of the universe with exotic pressure. From equation (24) we find that in case of stiff fluid (i.e., ρ = p) model, the deceleration parameter is positive, so in case (i) the model decelerates in the standard way.

V. CONCLUSION

In this paper, we have consider a five dimensional FRW space-time in scalar-tensor theory of gravitation defined by Brans and Dicke (1961) in the presence of perfect fluid distribution. The models obtained represents stiff fluid, disorder radiation, dust and false vacuum in a five dimensional flat FRW space time. It is defined that the model in each case is free from initial singularity. From figure-1 and figure-4, we have found that the volume scale factor of the universe increases with the growth of cosmic time t for all cases [i.e. case (i), case(ii), case(iii) and case(iv)]. Also from figure-2, 3, 5, 6, we obtained that the pressure and energy density in all the models[(i.e., case(i), case(ii) and case(iii)] diverges at t = 0 and vanish at $t\to\infty$ except case(iv) (i.e., false vacuum). Again from figure-7 we get the pressure is negative which shows that the universe is an accelerating mode. The models defined here will help us to understand Brans and Dicke cosmology in five dimension.

References

[1] C. Brans and R.H. Dicke: Mach's Principle and a Relativistic Theory of Gravitation, Physics, Vol. 124 (1961), pp-925-935.
[2] N.D. Birrell and P.C.W. Davies: Quantum Fields in Curved Space, Logica Pty Ltd, Australia (1984).
[3] H.J. Blome and W. Priester: Einstein-Friedmann-Kosmos und das Neutrino-Problem, Springer-Verlag , Vol. 71, (1984), pp 456-467.
[4] C.J. Hogan and M.J. Rees: Gravitational interaction of cosmic strings, Theoretical Astrophysics, Vol.311, (1984), pp-109-114.
[5] N. Kaiser and A. Stebbins: Microwave anisotropy due to cosmic strings, Theoretical Physics, Vol. 310 (1984), pp-391-393.
[6] K. Bichteler: On the Cauchy Problem of the Relativistic Boltzmann Equation, Physics, Vol.4 (1967), pp-352-364.
[7] D. Saez and V.J.Ballester: A Simple Coupling with Cosmological Implications, North Holand Physics Publishing Division, Vol. 113A (1986), pp. 467-460.
[8] C. Mathiazhagan and V. B. Johri: An inflationary universe in Brans-Dicke theory: a hopeful sign of theoretical estimation of the gravitational constant, The Institute of Physics, Vol.1 (1984), pp-29-32.
[9] Daile La and Paul J. Steinhardt: Extended Inflationary Cosmology, Physics, Vol. 62 (1989), pp-376-378.
[10] Paul J.Steinhardt and Frank S. Accetta: Hyperextended Inflation, Physics, Vol. 64 (1990), pp-2740-2743.
[11] A. Linde: Extended chaotic inflation and spatial variations of the gravitational constant, Physics, Vol. 238 (1990), pp-160-165.
[12] William J. Marciano: Time Variation of the Fundamental "Constants" and Kaluza-Klein Theories, Physics Rev, Vol. 52 (1984), pp-489-491.
[13] Edward Witten: Cosmic separation of phases, Physics Rev, Vol. 30 (1984), pp-270-285.
[14] Appelquist, T., et al.: Modern Klein Theories. Addison Wesley, Reading (1987).
[15] A. Chodos and S. Detweiler: Where has the Fifth Dimension Gone, The American Physical Society, Vol. 21 (1980), pp. 2167-2170.
[16] D.R.K. Reddy et al.: A higher-dimensional string cosmological model in Brans–Dicke theory of gravitation, Astrophys Space Sci, Vol. 310 (2007), pp-177-180.
[17] D.R.K. Reddy · R.L. Naidu:A Higher Dimensional Inflationary Universe in General Relativity, Int J Theor Phys, Vol. 47 (2008), pp-2339–2343.

[18] A. F. Bahrehbakhsh et al.:FRW cosmology from five dimensional vacuum Brans–Dicke theory, Springer Science, Vol. 43 (2011), pp-847–869.
[19] K.M. Singh and K. P. Singh: String cosmological models in the Brans-Dicke theory for five-dimensional space-time, Research in Astron. Astrophys, Vol. 12 (2012), pp-39–45.
[20] R.L. Naidu et al.: A five dimensional Kaluza-Klein bulk viscous string cosmological model in Brans-Dicke scalar-tensor theory of gravitation, Astrophys Space Sci, Vol. 347 (2013), pp-197–201.
[21] D.R.K. Reddy · G.V. Vijaya Lakshmi: Five dimensional radiating model in Brans-Dicke theory of gravitation, Astrophys Space Sci, Vol. 354 (2014), pp-1-14.
[22] V. U. M. Rao et al.: Five Dimensional FRW Cosmological Models in a Scalar-Tensor Theory of Gravitation, Astrophys Space Sci, Vol.164 (2015), pp. 1-5.
[23] T. Ramprasad et al.:Five dimensional FRW bulk viscous cosmological models in Brans–Dicke theory of gravitation, Astrophys Space Sci, Vol. 357:132 (2015), pp-1-4.
[24] D.R.K. Reddy et al.:Five dimensional minimally interacting holographic dark energy model in Brans–Dicke theory of gravitation, Astrophys Space Sci, Vol. 361:356 (2016), pp-1-6.
[25] G. P. Singh and Binaya K. Bishi:Bulk Viscous Cosmological Model in Brans-Dicke Theory with New Form of Time Varying Deceleration Parameter, Advances in High Energy Physics, Vol.2017 (2017), pp. 1-24.
[265]V.U.M.Rao et al.:Kaluza-Klein Holographic Cosmological Model in Brans-Dicke Theory of Gravitation, Advances in High Energy Physics, Vol. 117 (2017), pp. 383-393.
[27] Y. Aditya and D. R. K. Reddy: FRW type Kaluza–Klein modified holographic Ricci dark energymodels in Brans–Dicke theory of gravitation, Phys. J. C, Vol.78:619 (2018), pp. 1-19.
[28] Jianbo Lu et al.: The generalized Brans-Dicke theory and its cosmology, Phys. J. Plus, Vol.134: 318 (2019), pp. 1-10.
[29] Rami Ahmad El-Nabulsi: Five-dimensional Brans Dicke compactified universe dominated by a varying speed of light, Modern Physics Letters A, Vol.2050252 (2020), pp. 1-8.
[30] Shin’ichi Nojiri and Sergei D. Odintsov: Modified gravity with negative and positive powers of curvature: Unification of inflation and cosmic acceleration, The American Physical Society, Vol.D 68, (2003), pp. 1-10.