Characterization and Estimation of Area Biased Two Parameter Odoma Distribution with Application of Survival Times

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Abstract

In this paper, we study a new version of two parameter Odoma distribution namely Area biased two parameter Odoma distribution. The different structural properties of newly executed distribution have been obtained as the moments, reliability measures, order statistics and harmonic mean. The parameters of the proposed model have been estimated through the technique of maximum likelihood estimator and also its Fisher's Information matrix have been derived and discussed. Finally, a real life data set is considered to show that the area biased two parameter Odomadistribution has a better fit as compared to two parameter Odoma, one parameter Odoma, exponential and Lindley distributions.

Keywords: Weighteddistribution,Two parameterOdomadistribution, Reliability measures,Order statistics, Maximum likelihood estimator.

1 .Introduction

A new lifetime distribution namely two parameter Odomadistribution was introduced by Enogwe*et al.*(2020) enjoys a close distributional expression form, is a more flexible than the one parameter Odoma distribution. The proposed two parameter Odoma distribution is a special case of one parameter Odoma distribution. The one parameter Odoma distribution namely Odoma distribution and its Application was introduced by C.C.Odom and M.A. Ijomah (2019) and discuss its various structural properties. Various statistical properties of the two parameter Odomadistribution including its moments (raw and central), skewness, kurtosis, index of dispersion, moment generating function, characteristic function, order statistics, stochastic ordering,mean deviation, Bonferroni and Lorenz curves, Renyi entropy, reliability measures, quantile function and mean residual life function have been obtained and presented. For estimating the parameters of two parameter Odoma distribution, the technique of maximum likelihood estimator have been employed and also its asymptotic confidence interval have been discussed. The goodness of fit of two parameter Odoma distribution has been illustrated by using the two real data sets and the fit has been found goodin comparison with some well-known distributions such as one parameter Odoma, two parameter Lindley, two parameter Akash, two parameter Sujatha, two parameter Rama and two parameter Pranav distributions.

The probability density function of two parameterOdomadistribution (TPOD) with parameters α and β is given by

$$f(x;\alpha,\beta)=\frac{\alpha^5}{2\left(\alpha^5\beta+\alpha^3+24\right)}\left(2x^4+\alpha x^2+2\alpha\beta\right)e^{-\alpha x};\ x>0,\alpha>0,\beta>0 \tag{1}$$

and the cumulative distribution function of two parameterOdomadistribution is given by

$$F(x;\alpha,\beta)=1-\left(1+\frac{2\alpha^2x^2\left(\alpha^2x^2+4\alpha x+12\right)+\alpha x^2\left(\alpha^4x+2\alpha^3+48\right)}{2\left(\alpha^5\beta+\alpha^3+24\right)}\right)e^{-\alpha x};x>0,\alpha>0,\beta>0 \tag{2}$$

2. Area Biased Two Parameter OdomaDistribution(ABTPOD)

The study of lifetime data and distributions is considered to be important in research areas such as engineering, biomedical sciences, environmental sciences and actuarial science. The lifetime data is random phenomena may come from diversified fields, so for obtaining decisions from lifetime data its modeling becomes important. Various probability models have been applied to lifetime data. One such model is of weighted model. The theory of weighted distributions arises when the observations generated from a stochastic process are not given equal chances of being recorded, instead they are recorded according to some weight function. The idea of weighted distributions was first introduced by Fisher (1934) to model the ascertainment bias, Later Rao (1965) introduced and formulated in a general way for modeling statistical data, because the classical distributions was found to be inappropriate.The weighted distributions are useful in distribution theory because it provides an approach to deal with model specification and data interpretation problems.Weighted distributions also takesinto account the method of ascertainment, by adjusting the probabilities of the actual occurrence of events to arrive at a specification of the probabilities of those events as observed and recorded. The weighted distributions are also useful because it provides an understanding of the existing standard probability distributions and it provides methods for extending existing standard probability distributions for modeling lifetime data due to introduction of additional parameter in the model which creates flexibility in their nature. The weighted distribution reduces to length biased distribution when the weight function considers only the length of units. The concept of length biased distribution was first introduced by Cox (1969) and Zelen (1974). More generally, when the sampling mechanism selects units with probability proportional to measure of the unit size, resulting distribution is called size-biased distribution. The statistical interpretation of length biased distribution was originally identified by Cox (1962) in the context of renewal theory. A lot of work has been carried out by various researchers to characterize the relationship between classical distribution and their area biased versions. Ade *et al.* (2021) have discussed the characterization and estimation of area biased quasi Akash distribution. Ade *et al.* (2020) also studied the Area biased generalized uniform distribution and discuss its several structural properties.Osowole*et al.* (2020) have obtained the Area biased quasi transmuted uniform distribution. Subramanian and Shenbagaraja (2020) have derived and discussed the length biased quasi sujathadistribution with properties and applications to bladder cancer data. Bashir and Mahmood (2019) have introduced the multivariate area biased Lindley distribution with properties and applications.Oluwafemi and Olalekan (2017) presented the length and area biased exponentiated weibull distribution based on forest inventories. Alidamat and Al-omari (2021) discussed on the length biased two parameter Mirra distribution with applications of engineering data. Beghriche and Zeghdoudi(2019) proposed a size biased gamma Lindley distribution. Mudasir and Ahmad (2015) obtained the length biased version of Nakagami distribution. Ganaie and Rajagopalan (2020) discussed on the length biased quasi exponential distribution with properties and applications. Kersey and Oluyede (2012) presented the length biased inverse Weibull distribution. Mir *et al.* (2013) have derived and discussed the length biased beta distribution of first kind. Recently, Elangovan and Mohanasundari (2019) have derived the area biased Aradhana distribution with applications, which shows more flexible and reliable than the classical distribution.

To introduce the weighted distribution, let us consider the non-negative random variable $X$with its natural probability density function $f(x)$.Let its non-negative weightfunction be $w(x)$, then the probability density function of the weighted random variable X_w is given by

$$f_w(x) = \frac{w(x) f(x)}{E(w(x))}, x > 0.$$

assuming that $E(w(x)) = \int w(x) f(x) dx < \infty$.i.e the first moment of $w(x)$ exists

In this paper, we have to obtain the area biased version of two parameter Odomadistribution. Soby taking consequently at $w(x) = x^2$, the resulting distribution is termed as area biased distribution with its probability density function given by

$$f_a(x) = \frac{x^2 f(x)}{E(x^2)} \tag{3}$$

Where $E(x^2) = \int_0^\infty x^2 f(x) dx$

$$E\left(x^2\right) = \frac{1440 + 24\alpha^3 + 4\beta\alpha^6}{\alpha^2\left(2\left(\alpha^5\beta + \alpha^3 + 24\right)\right)} \tag{4}$$

Substitute equations (1) and (4) in equation (3), we will obtain the probability density function of Area biased two parameterOdomadistribution

$$f_a(x) = \frac{\alpha^7}{\left(1440 + 24\alpha^3 + 4\beta\alpha^6\right)} x^2\left(2x^4 + \alpha x^2 + 2\alpha\beta\right) e^{-\alpha x} \tag{5}$$

and the cumulative distribution function of Area biased two parameter Odoma distribution can be obtained as

$$F_a(x) = \int_0^x f_a(x)\,dx$$

$$F_a(x) = \int_0^x \left(\frac{\alpha^7}{1440 + 24\alpha^3 + 4\beta\alpha^6}\right) x^2\left(2x^4 + \alpha x^2 + 2\alpha\beta\right) e^{-\alpha x}\,dx$$

$$F_a(x) = \left(\frac{\alpha^7}{1440 + 24\alpha^3 + 4\beta\alpha^6}\right) \int_0^x x^2\left(2x^4 + \alpha x^2 + 2\alpha\beta\right) e^{-\alpha x}\,dx$$

$$F_a(x) = \left(\frac{\alpha^7}{1440 + 24\alpha^3 + 4\beta\alpha^6}\right) \left(2\int_0^x x^6 e^{-\alpha x}\,dx + \alpha \int_0^x x^4 e^{-\alpha x}\,dx + 2\alpha\beta \int_0^x x^2 e^{-\alpha x}\,dx\right)$$

Put $\alpha x = t \Rightarrow \alpha dx = dt \Rightarrow dx = \frac{dt}{\alpha}$

When $x \to x, t \to \alpha x$ and When $x \to 0, t \to 0$, Also $x = \frac{t}{\alpha}$

After simplification, we obtain the cumulative distribution function of Area biased two parameterOdomadistribution as

$$F_a(x) = \left(\frac{1}{1440 + 24\alpha^3 + 4\beta\alpha^6}\right) \left(2\gamma(7, \alpha x) + \alpha^3 \gamma(5, \alpha x) + 2\beta\alpha^5 \gamma(3, \alpha x)\right) \quad (6)$$

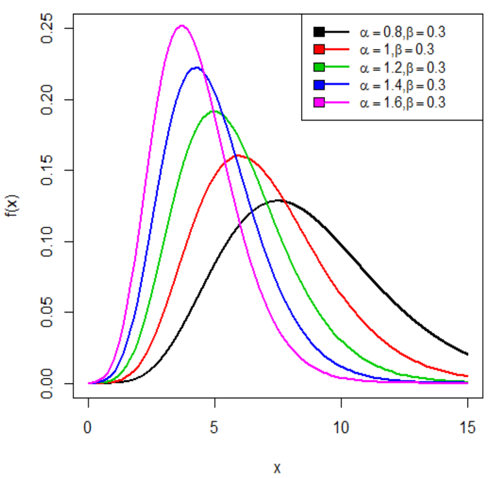

Fig.1:Pdf plot of Area Biased Two Parameter Odoma distribution

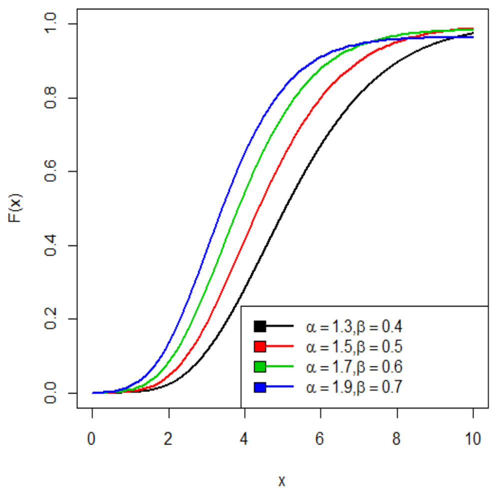

Fig.2 Cdf plot of Area Biased Two Parameter Odoma distribution

3. ReliabilityAnalysis

In thissection, we will discuss the survival function, hazard rate and reverse hazard rate functionsof the Area biased two parameterOdomadistribution.

a) Survival function

The survival function of Area biased two parameterOdomadistribution can be obtained as

$$S(x) = 1 - F_a(x)$$

$$S(x) = 1 - \frac{1}{\left(1440 + 24\alpha^3 + 4\beta\alpha^6\right)}\left(\left(2\gamma(7, \alpha x) + \alpha^3\gamma(5, \alpha x) + 2\beta\alpha^5\gamma(3, \alpha x)\right)\right)$$

b) Hazard function

The hazard function is also known as hazard rate or failure rate or force of mortality and is given by

$$h(x) = \frac{f_a(x)}{S(x)}$$

$$h(x) = \frac{\alpha^7 x^2\left(\left(2x^4 + \alpha x^2 + 2\alpha\beta\right)\right) e^{-\alpha x}}{\left(1440 + 24\alpha^3 + 4\beta\alpha^6\right) - \left(2\gamma(7, \alpha x) + \alpha^3\gamma(5, \alpha x) + 2\beta\alpha^5\gamma(3, \alpha x)\right)}$$

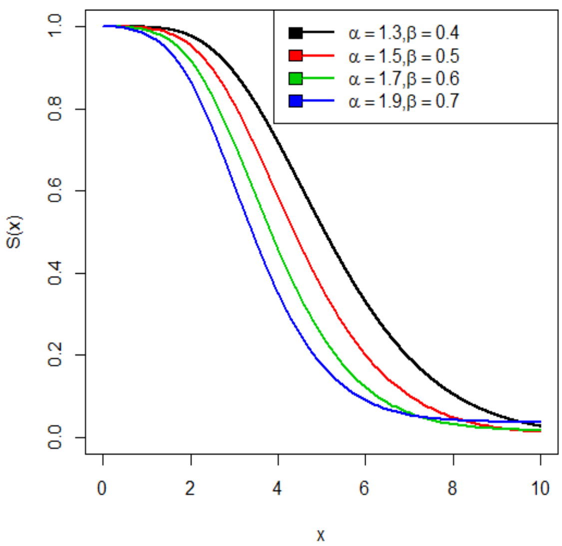

Fig.3:Survival plot of Area Biased Two Parameter Odoma distribution

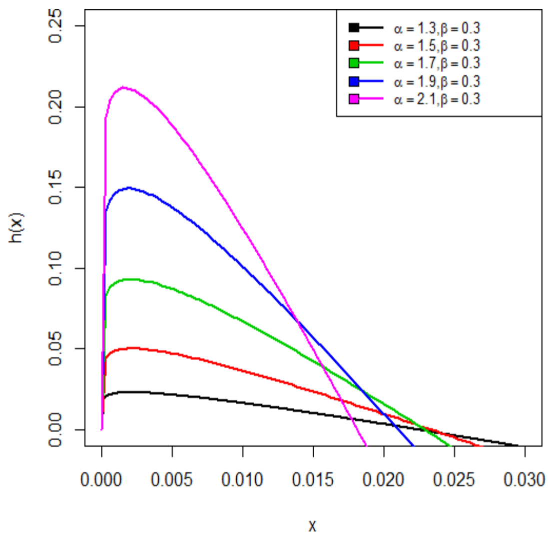

Fig.4:Hazard plot of Area Biased Two Parameter Odoma distribution

c) Reverse hazard function

The reverse hazard function of Area biased two parameterOdomadistribution can be obtained as

$$h_r(x) = \frac{f_a(x)}{F_a(x)}$$

$$h_r(x) = \frac{\alpha^7 x^2 \left(2x^4 + \alpha x^2 + 2\alpha\beta\right) e^{-\alpha x}}{\left(2\gamma(7, \alpha x) + \alpha^3 \gamma(5, \alpha x) + 2\beta\alpha^5 \gamma(3, \alpha x)\right)}$$

4. Structural Properties

In this section, we will investigate various structural properties of Area biased two parameterOdomadistribution especially its moments, harmonic mean, moment generating function and characteristic function.

Moments

Suppose X is a random variable following Area biased two parameterOdomadistributionwith parameters*α*and*β,* then the r^{th} order moment $E(X^r)$ of X can be obtained as

$$E(X^r) = \mu_r' = \int_0^\infty x^r f_a(x)dx$$

$$= \int_0^\infty x^r \frac{\alpha^7}{\left(1440 + 24\alpha^3 + 4\beta\alpha^6\right)} x^2 \left(2x^4 + \alpha x^2 + 2\alpha\beta\right) e^{-\alpha x} dx$$

$$= \frac{\alpha^7}{\left(1440 + 24\alpha^3 + 4\beta\alpha^6\right)} \int_0^\infty x^{r+2} \left(2x^4 + \alpha x^2 + 2\alpha\beta\right) e^{-\alpha x} dx$$

$$= \frac{\alpha^7}{\left(1440 + 24\alpha^3 + 4\beta\alpha^6\right)} \left(2\int_0^\infty x^{(r+7)-1} e^{-\alpha x} dx + \alpha \int_0^\infty x^{(r+5)-1} e^{-\alpha x} dx + 2\alpha\beta \int_0^\infty x^{(r+3)-1} e^{-\alpha x} dx\right) \quad (7)$$

After simplification of equation (7), we obtain

$$E(X^r) = \mu_r' = \frac{2\Gamma(r+7) + \alpha^3\Gamma(r+5) + 2\beta\alpha^6\Gamma(r+3)}{\alpha^r\left(1440 + 24\alpha^3 + 4\beta\alpha^6\right)} \quad (8)$$

Substituting r = 1,2, 3 and 4 in equation (8), we will obtain the first four momentsof Area biased two parameterOdomadistribution.

$$E(X) = \frac{10080 + 120\alpha^3 + 12\beta\alpha^6}{\alpha\left(1440 + 24\alpha^3 + 4\beta\alpha^6\right)}$$

$$E(X^2) = \frac{80640 + 720\alpha^3 + 48\beta\alpha^6}{\alpha^2\left(1440 + 24\alpha^3 + 4\beta\alpha^6\right)}$$

$$E(X^3) = \frac{725760 + 5040\alpha^3 + 240\beta\alpha^6}{\alpha^3\left(1440 + 24\alpha^3 + 4\beta\alpha^6\right)}$$

$$E(X^4) = \frac{7257600 + 40320\alpha^3 + 1440\beta\alpha^6}{\alpha^4\left(1440 + 24\alpha^3 + 4\beta\alpha^6\right)}$$

$$\text{Variance} = \frac{80640 + 720\alpha^3 + 48\beta\alpha^6}{\alpha^2\left(1440 + 24\alpha^3 + 4\beta\alpha^6\right)} - \left(\frac{10080 + 120\alpha^3 + 12\beta\alpha^6}{\alpha\left(1440 + 24\alpha^3 + 4\beta\alpha^6\right)}\right)^2$$

$$S.D(\sigma) = \sqrt{\frac{80640 + 720\alpha^3 + 48\beta\alpha^6}{\alpha^2\left(1440 + 24\alpha^3 + 4\beta\alpha^6\right)} - \left(\frac{10080 + 120\alpha^3 + 12\beta\alpha^6}{\alpha\left(1440 + 24\alpha^3 + 4\beta\alpha^6\right)}\right)^2}$$

Harmonic mean

The harmonic mean of the proposed model can be obtained as

$$H.M = E\left(\frac{1}{x}\right) = \int_0^\infty \frac{1}{x} f_a(x)dx$$

$$= \int_0^\infty \frac{\alpha^7}{\left(1440 + 24\alpha^3 + 4\beta\alpha^6\right)} x\left(2x^4 + \alpha x^2 + 2\alpha\beta\right) e^{-\alpha x} dx$$

$$= \frac{\alpha^7}{\left(1440 + 24\alpha^3 + 4\beta\alpha^6\right)} \int_0^\infty x\left(2x^4 + \alpha x^2 + 2\alpha\beta\right) e^{-\alpha x} dx$$

$$= \frac{\alpha^7}{\left(1440+24\alpha^3+4\beta\alpha^6\right)}\left(2\int_0^\infty x^5 e^{-\alpha x}dx+\alpha\int_0^\infty x^3 e^{-\alpha x}dx+2\alpha\beta\int_0^\infty x\, e^{-\alpha x}dx\right)$$

$$= \frac{\alpha^7}{\left(1440+24\alpha^3+4\beta\alpha^6\right)}\left(2\int_0^\infty x^{6-1} e^{-\alpha x}dx+\alpha\int_0^\infty x^{4-1} e^{-\alpha x}dx+2\alpha\beta\int_0^\infty x^{3-2} e^{-\alpha x}dx\right) \quad (9)$$

After simplification of equation (9), we obtain

$$H.M = \frac{\alpha^7}{\alpha^6\left(1440+24\alpha^3+4\beta\alpha^6\right)}\left(240+6\alpha^3+4\beta\alpha^5\right)$$

Moment Generating Function and Characteristic Function

Suppose the random variable X following Area biased two parameter Odoma distribution with parameters α and β, then the MGF of X can be obtained as

$$M_X(t)=E\left(e^{tx}\right)=\int_0^\infty e^{tx} f_a(x)\,dx$$

Using Taylor's series, we obtain

$$=\int_0^\infty\left(1+tx+\frac{(tx)^2}{2!}+\ldots\right)f_a(x)dx$$

$$=\int_0^\infty\sum_{j=0}^\infty\frac{t^j}{j!}x^j f_a(x)dx$$

$$=\sum_{j=0}^\infty\frac{t^j}{j!}\mu_j'$$

$$=\sum_{j=0}^\infty\frac{t^j}{j!}\left(\frac{2\Gamma(j+7)+\alpha^3\Gamma(j+5)+2\beta\alpha^6\Gamma(j+3)}{\alpha^j\left(1440+24\alpha^3+4\beta\alpha^6\right)}\right)$$

$$\Rightarrow M_X(t)=\frac{1}{\left(1440+24\alpha^3+4\beta\alpha^6\right)}\sum_{j=0}^\infty\frac{t^j}{j!\alpha^j}\left(2\Gamma(j+7)+\alpha^3\Gamma(j+5)+2\beta\alpha^6\Gamma(j+3)\right)$$

Similarly, the characteristic function of Area biased two parameterOdomadistribution can be obtained as

$$\varphi_X(t) = M_X(it)$$

$$M_X(it) = \left(\frac{1}{1440+24\alpha^3+4\beta\alpha^6}\right)\sum_{j=0}^{\infty}\frac{it^j}{j!\alpha^j}\left(2\Gamma(j+7)+\alpha^3\Gamma(j+5)+2\beta\alpha^6\Gamma(j+3)\right)$$

5. Order Statistics

Order statistics plays a key role in several aspects of statistical inference. Order statistics has a lot of applications in the field of reliability and life testing.Let$X_{(1)}$, $X_{(2)}$,…, $X_{(n)}$be the order statistics of a random sample $X_1, X_2,…,X_n$ from a continuous population with probability density function$f_x(x)$andcumulative distribution function with $F_X(x)$, then the probability density function of r^{th} order statistics $X_{(r)}$is given by

$$f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!} f_X(x)\left(F_X(x)\right)^{r-1}\left(1-F_X(x)\right)^{n-r} \tag{10}$$

Substituting the values of equation (5) and (6) in equation (10), we will obtain the probability density functionofr^{th}order statistics ofArea biased two parameterOdomadistribution

$$f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!}\left(\frac{\alpha^7}{\left(1440+24\alpha^3+4\beta\alpha^6\right)}x^2\left(2x^4+\alpha x^2+2\alpha\beta\right)e^{-\alpha x}\right)$$

$$\times\left(\left(\frac{1}{1440+24\alpha^3+4\beta\alpha^6}\right)\left(2\gamma(7,\alpha x)+\alpha^3\gamma(5,\alpha x)+2\beta\alpha^5\gamma(3,\alpha x)\right)\right)^{r-1}$$

$$\times\left(1-\left(\frac{1}{1440+24\alpha^3+4\beta\alpha^6}\right)\left(2\gamma(7,\alpha x)+\alpha^3\gamma(5,\alpha x)+2\beta\alpha^5\gamma(3,\alpha x)\right)\right)^{n-r}$$

Therefore, the probability density function of higherorder statistic $X_{(n)}$ of Area biased two parameterOdomadistribution can be obtained as

$$f_{x(n)}(x) = \left(\frac{n\alpha^7}{1440+24\alpha^3+4\beta\alpha^6}\right)x^2\left(2x^4+\alpha x^2+2\alpha\beta\right)e^{-\alpha x}$$
$$\times\left(\frac{1}{\left(1440+24\alpha^3+4\beta\alpha^6\right)}\left(2\gamma(7,\alpha x)+\alpha^3\gamma(5,\alpha x)+2\beta\alpha^5\gamma(3,\alpha x)\right)\right)^{n-1}$$

and the probability density function of first order statistic $X_{(1)}$ of Area biased two parameter Odomadistribution can be obtained as

$$f_{x(1)}(x) = \left(\frac{n\alpha^7}{1440+24\alpha^3+4\beta\alpha^6}\right)x^2\left(2x^4+\alpha x^2+2\alpha\beta\right)e^{-\alpha x}$$
$$\times\left(1-\frac{1}{\left(1440+24\alpha^3+4\beta\alpha^6\right)}\left(2\gamma(7,\alpha x)+\alpha^3\gamma(5,\alpha x)+2\beta\alpha^5\gamma(3,\alpha x)\right)\right)^{n-1}$$

6. Likelihood Ratio Test

Consider the random sample $X_1, X_2, \ldots, X_n$of size n drawn from the Area biased two parameterOdomadistribution. We set up the hypothesis for testing.

$$H_o : f(x) = f(x;\alpha,\beta) \quad \text{against} \quad H_1 : f(x) = f_a(x;\alpha,\beta)$$

In order to investigate, whether the random sample of size $n$comes from the two parameterOdomadistribution or Area biased two parameterOdomadistribution, the following test statistic procedure is used.

$$\Delta = \frac{L_1}{L_o} = \prod_{i=1}^{n}\frac{f_a(x;\alpha,\beta)}{f(x;\alpha,\beta)}$$

$$\Delta = \frac{L_1}{L_o} = \prod_{i=1}^{n}\left(\frac{\alpha^2 x_i^2\left(2\left(\alpha^5\beta+\alpha^3+24\right)\right)}{\left(1440+24\alpha^3+4\beta\alpha^6\right)}\right)$$

$$\Delta = \frac{L_1}{L_o} = \left(\frac{\alpha^2\left(2\left(\alpha^5\beta+\alpha^3+24\right)\right)}{\left(1440+24\alpha^3+4\beta\alpha^6\right)}\right)^n \prod_{i=1}^{n} x_i^2$$

We should refuse to accept the null hypothesis, if

$$\Delta = \left(\frac{\alpha^2 \left(2\left(\alpha^5 \beta + \alpha^3 + 24 \right) \right)}{\left(1440 + 24\alpha^3 + 4\beta\alpha^6 \right)} \right)^n \prod_{i=1}^{n} x_i^2 > k$$

Or equivalently, we should reject the null hypothesis, if

$$\Delta^* = \prod_{i=1}^{n} x_i^2 > k \left(\frac{\left(1440 + 24\alpha^3 + 4\beta\alpha^6 \right)}{\alpha^2 \left(2\left(\alpha^5 \beta + \alpha^3 + 24 \right) \right)} \right)^n$$

$$\Delta^* = \prod_{i=1}^{n} x_i^2 > k^*, \text{ Where } k^* = k \left(\frac{\left(1440 + 24\alpha^3 + 4\beta\alpha^6 \right)}{\alpha^2 \left(2\left(\alpha^5 \beta + \alpha^3 + 24 \right) \right)} \right)^n$$

For large sample size *n*, *2log Δ* is distributed as chi-square distribution with one degree of freedom and also *p*-value is obtained from the chi-square distribution. Thus, we refuse to accept the null hypothesis, when the probability value is given by $p\left(\Delta^* > \theta^*\right)$, Where $\theta^* = \prod_{i=1}^{n} x_i^2$ is less than a specified level of significance and $\prod_{i=1}^{n} x_i^2$ is the observed value of the statistic Δ^*.

7. Bonferroni and Lorenz Curves

The Bonferroni and Lorenz curves are employed in various areas like reliability, medicine, insurance and demography. The bonferroni and Lorenz curves are given by

$$B(p) = \frac{1}{p\mu_1'} \int_0^q x f(x) dx$$

and $$L(p) = pB(p) = \frac{1}{\mu_1'} \int_0^q x f(x) dx$$

Where $$\mu_1' = E(X) = \frac{10080 + 120\alpha^3 + 12\beta\alpha^6}{\alpha\left(1440 + 24\alpha^3 + 4\beta\alpha^6 \right)}$$ and $q = F^{-1}(p)$

$$B(p) = \frac{\alpha\left(1440+24\alpha^3+4\beta\alpha^6\right)}{p\left(10080+120\alpha^3+12\beta\alpha^6\right)} \int_0^q \frac{\alpha^7}{\left(1440+24\alpha^3+4\beta\alpha^6\right)} x^3\left(2x^4+\alpha x^2+2\alpha\beta\right) e^{-\alpha x} dx$$

$$B(p) = \frac{\alpha^8}{p\left(10080+120\alpha^3+12\beta\alpha^6\right)} \int_0^q x^3\left(2x^4+\alpha x^2+2\alpha\beta\right) e^{-\alpha x} dx$$

$$B(p) = \frac{\alpha^8}{p\left(10080+120\alpha^3+12\beta\alpha^6\right)} \left(2\int_0^q x^{8-1} e^{-\alpha x} dx + \alpha \int_0^q x^{6-1} e^{-\alpha x} dx + 2\alpha\beta \int_0^q x^{4-1} e^{-\alpha x} dx\right)$$

After simplification, we get

$$B(p) = \frac{\alpha^8}{p\left(10080+120\alpha^3+12\beta\alpha^6\right)} \left(2\gamma(8,\alpha q) + \alpha\gamma(6,\alpha q) + 2\alpha\beta\gamma(4,\alpha q)\right)$$

$$L(p) = \frac{\alpha^8}{\left(10080+120\alpha^3+12\beta\alpha^6\right)} \left(2\gamma(8,\alpha q) + \alpha\gamma(6,\alpha q) + 2\alpha\beta\gamma(4,\alpha q)\right)$$

8. Maximum Likelihood Estimator and Fisher's Information Matrix

In this section, the parameter estimation of Area biased two parameter Odoma distributionhas been obtained by using the technique of maximum likelihood estimator and also its Fisher's information matrix have been discussed. Let $X_1, X_2, \ldots, X_n$ be a random sample of size *n*from the Area biased two parameterOdomadistribution, then the likelihood function can be written as

$$L(x) = \prod_{i=1}^{n} f_a(x)$$

$$L(x) = \prod_{i=1}^{n} \left(\frac{\alpha^7}{\left(1440+24\alpha^3+4\beta\alpha^6\right)} x^2\left(2x_i^4+\alpha x_i^2+2\alpha\beta\right) e^{-\alpha x_i} \right)$$

$$L(x) = \frac{\alpha^{7n}}{\left(1440 + 24\alpha^3 + 4\beta\alpha^6\right)^n} \prod_{i=1}^{n} \left(x_i^2 \left(2x_i^4 + \alpha x_i^2 + 2\alpha\beta \right) e^{-\alpha x_i} \right)$$

The log likelihood function is given by

$$\log L = 7n \log\alpha - n\log\left(1440 + 24\alpha^3 + 4\beta\alpha^6\right) + 2\sum_{i=1}^{n} \log x_i + \sum_{i=1}^{n} \log\left(2x_i^4 + \alpha x_i^2 + 2\alpha\beta\right) - \alpha \sum_{i=1}^{n} x_i \qquad (12)$$

The maximum likelihood estimate of α and βcan be obtained by differentiating the log

likelihood equation (12) with respect to parameters$\alpha$and β. We obtain the normal equations

as

$$\frac{\partial \log L}{\partial \alpha} = \frac{7n}{\alpha} - n\left(\frac{72\alpha^2 + 24\beta\alpha^5}{\left(1440 + 24\alpha^3 + 4\beta\alpha^6\right)}\right) + \sum_{i=1}^{n}\left(\frac{\left(x_i^2 + 2\beta\right)}{\left(2x_i^4 + \alpha x_i^2 + 2\alpha\beta\right)}\right) - \sum_{i=1}^{n} x_i = 0$$

$$\frac{\partial \log L}{\partial \beta} = -n\left(\frac{4\alpha^6}{\left(1440 + 24\alpha^3 + 4\beta\alpha^6\right)}\right) + \sum_{i=1}^{n}\left(\frac{2\alpha}{\left(2x_i^4 + \alpha x_i^2 + 2\alpha\beta\right)}\right) = 0$$

The above likelihood equations are too complicated to solve it algebraically.Therefore, we use the numerical technique like Newton-Raphson method for estimating the parameters of the proposed distribution.

We use the asymptotic normality results to obtain the confidence interval. We have that if$\hat{\lambda} = (\hat{\alpha}, \hat{\beta})$ denotes the MLE of $\lambda = (\alpha, \beta)$. We can state the result as follows:

$$\sqrt{n}(\hat{\lambda} - \lambda) \rightarrow N_2(0, I^{-1}(\lambda))$$

Where $I(\lambda)$is the Fisher's Informationmatrix, i.e.,

$$I(\lambda) = -\frac{1}{n}\begin{pmatrix} E\left(\frac{\partial^2 \log L}{\partial \alpha^2}\right) & E\left(\frac{\partial^2 \log L}{\partial \alpha \partial \beta}\right) \\ E\left(\frac{\partial^2 \log L}{\partial \beta \partial \alpha}\right) & E\left(\frac{\partial^2 \log L}{\partial \beta^2}\right) \end{pmatrix}$$

Here we define

$$E\left(\frac{\partial^2 \log L}{\partial \alpha^2}\right) = -\frac{7n}{\alpha^2} - n\left(\frac{\left(1440+24\alpha^3+4\beta\alpha^6\right)\left(144\alpha+120\beta\alpha^4\right)-\left(72\alpha^2+24\beta\alpha^5\right)\left(72\alpha^2+24\beta\alpha^5\right)}{\left(1440+24\alpha^3+4\beta\alpha^6\right)^2}\right) - \sum_{i=1}^{n}\left(\frac{\left(x_i^2+2\beta\right)^2}{\left(2x_i^4+\alpha x_i^2+2\alpha\beta\right)^2}\right)$$

$$E\left(\frac{\partial^2 \log L}{\partial \beta^2}\right) = n\left(\frac{\left(4\alpha^6\right)^2}{\left(1440+24\alpha^3+4\beta\alpha^6\right)^2}\right) - \sum_{i=1}^{n}\left(\frac{4\alpha^2}{\left(2x_i^4+\alpha x_i^2+2\alpha\beta\right)^2}\right)$$

$$E\left(\frac{\partial^2 \log L}{\partial \alpha\, \partial \beta}\right) = -n\left(\frac{\left(1440+24\alpha^3+4\beta\alpha^6\right)\left(24\alpha^5\right)-\left(4\alpha^6\right)\left(72\alpha^2+24\beta\alpha^5\right)}{\left(1440+24\alpha^3+4\beta\alpha^6\right)^2}\right) + \sum_{i=1}^{n}\left(\frac{2\left(2x_i^4+\alpha x_i^2+2\alpha\beta\right)-2\alpha\left(x_i^2+2\beta\right)}{\left(2x_i^4+\alpha x_i^2+2\alpha\beta\right)^2}\right)$$

Since λ being unknown, we estimate $I^{-1}(\lambda)$ by $I^{-1}(\hat{\lambda})$ and this can be used to obtain asymptotic confidence intervalsfor α and β.

9. Application

In this section, we have used a real life data set in Area biased two parameter Odoma distribution to show that the Area biased two parameterOdomadistribution can be a better model than the two parameterOdoma, one parameter Odoma, exponential and Lindley distributions. The real life data set is given below in table 1.

The real lifetime data set corresponds to survival time in weeks of 44 rats which were given dosages of Cytoxan at a concentration of 60 mg/kg once weekly supplied by Professor S.C. Choi and was applied by McLachlan, Lawoko and Ganesalingam (1982). This data set is available on book Mixiture Models by G. J. McLachlan and K. E. Basford on page number 121.

Table 1: Data regarding survival time of 44 rats in weeks

7.25	5.25	6.75	5.5	7.25	6.75	7.5	6.5	7.5
8.75	12.25	12.5	6.5	7	6.5	7	10	5.5
10	7.25	4	7.5	9	7	4.75	7	8.75
7.5	7.75	7.5	7.25	16	14.5	6.75	16	12.5
18	5	7.5	6.5					

In order to estimate the model comparison criterion values, the unknown parameters are also estimated through the R software. In order to compare the Area biased two parameter Odomadistribution with two parameter Odom, one parameter Odoma, exponential and Lindley distributions, we are using the criterion values*AIC*(Akaike Information Criterion), *AICC* (Akaike Information Criterion Corrected), *BIC* (Bayesian Information Criterion) and -*2logL*. The better distribution is which corresponds to lesser values of AIC, AICC, BIC and -2logL. The following formulas are applied for the estimation of criterion values.

$$AIC = 2k - 2\log L \qquad BIC = k\log n - 2\log L \qquad \text{and} \qquad AICC = AIC + \frac{2k(k+1)}{n-k-1}$$

Where$k$is the number of parameters in the statistical model,n is the sample size and $-2\log L$is the maximized value of the log-likelihood function under the considered model.

Table 2: Performance of Distributions

Distribution	MLE	S.E	-2logL	AIC	BIC	AICC
Area Biased Two Parameter Odoma	$\hat{\alpha} = 0.84700361$ $\hat{\beta} = 0.00100000$	$\hat{\alpha} = 0.01590487$ $\hat{\beta} = 0.00156873$	211.8292	215.8292	219.3976	216.1218
Two Parameter Odoma	$\hat{\alpha} = 0.60429286$ $\hat{\beta} = 0.00100000$	$\hat{\alpha} = 0.01679397$ $\hat{\beta} = 0.01002369$	217.2474	221.2474	224.8157	221.5400
Odoma	$\hat{\alpha} = 0.60238962$	$\hat{\alpha} = 0.04014426$	217.5103	219.5103	221.2945	219.6055
Exponential	$\hat{\alpha} = 0.12138206$	$\hat{\alpha} = 0.01829779$	273.5775	275.5775	277.3617	275.6727
Lindley	$\hat{\alpha} = 0.22080648$	$\hat{\alpha} = 0.02373245$	251.8082	253.8082	255.5924	253.9034

From table 2, it has been observed from the results that the Area biased two parameter Odoma distribution have the lesser AIC, BIC, AICC and -2logL values as compared to the two parameter Odoma, one parameter Odoma, exponential and Lindley distributions, which concluded that the Area biased two parameter Odomadistribution leads to a better fit than the two parameter Odoma, one parameter Odoma, exponential and Lindley distributions.

10. Conclusion

In the present study, we have introduced a new generalization of two parameterOdoma distribution known as Area biased two parameterOdoma distribution. The subject distribution is generated by using the Area-biased technique and the parameters have been obtained through the technique of maximum likelihood estimator. The different statistical properties of the proposed distribution as moments, reliability measures, order statistics, Bonferroni and Lorenz curves have been derived and discussed. The new distribution with its applications in real life time data has been fitted. Finally, the results are compared over two parameter Odoma, one parameter Odoma, exponential and Lindley distributions and it is found from the results that the Area biased two parameter Odoma distribution fits better than the two parameter Odoma, one parameter Odoma, exponential and Lindley distributions.

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