Length biased OM distribution with properties and Applications to Survival Times

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Abstract

In this paper, a new version of OM distribution namely length biased OM distribution is proposed and studied. The density function and its behavior, moments, survival functions and hazard rate, reverse hazard rate, order statistics, entropies and likelihood ratio test have been discussed and studied. The parameters of this distribution are estimated by the method of moments and the maximum likelihood estimation method. Finally, an application of the model to a real life data set is presented and compared with other distributions.

Keywords: *length biased distribution, OM distribution, Entropies, Order statistics, Maximum likelihood estimation*

1. INTRODUCTION

The weighted distributions are used as a tool in selection of appropriate models for observed data, especially when samples are drawn without a proper frame. Weighted distributions have been employed in wide variety applications in reliability and survival analysis, meta-analysis, analysis of family data, ecology and forestry. The concept of weighted distributions was first emerged by Fisher (1934) is a traceable work in respect of his studies on how methods of ascertainment can affect the form of distribution of recorded observations. Later it was introduced and formulated in a more general way by Rao (1965) with respect to modelling statistical data where the routine practice of using standard distributions for the purpose was dismissed as inappropriate. The concept of length biased sampling was first introduced by Cox (1969) and Zelen (1974). The weighted distribution reduces to length biased distribution when the weight function considers only the length of the units. This concept is found in various applications in biomedical area such as survival analysis, family history and disease, intermediate events and latency period of AIDS due to blood transmission. There are various good sources which provide the detailed description of length biased distributions. Many newly introduced distributions along with their length biased versions exist in literature whose statistical behaviour is extensively studied during decades. Reyad et al. (2013) discussed on the length-biased weighted Frechet distribution with Properties and estimation. Seenoi et al. (2014) discussed on the length-biased exponentiated inverted Weibull distribution with various properties and applications. Modi and Gill (2015), obtained the length biased version of weighted Maxwell distribution with various statistical properties. Reyad et al. (2017), obtained the length biased weighted Frechet distribution with properties and estimation. Karimi and Nasiri (2018) discussed the length biased weighted Lomax distribution in the presence of outliers. Rather and Subramanian (2018), discussed on length biased Sushila distribution with properties and applications. Recently, Subramanian and Rather (2020) studied a new extension of Shanker distribution with real life data.

2. LENGTH BIASED OM (LBO) DISTRIBUTION

A new one parameter lifetime distribution named as OM distribution was introduced by Shanker and Shukla (2018).The probability density function (pdf) of OM distribution is given by

$$f(x;\theta)=\frac{\theta^5}{\theta^4+4\theta^3+12\theta^2+24\theta+24}(1+x)^4 e^{-\theta x};\ x>0,\theta>0 \quad (1)$$

and its cumulative distribution function (cdf) is given by

$$F(x;\theta)=1-\left[\frac{(1+x)^4\theta^4+4(1+x)^3\theta^3+12(1+x)^2\theta^2+24(1+x)\theta+24}{\theta^4+4\theta^3+12\theta^2+24\theta+24}\right]e^{-\theta x};\ x>0,\theta>0 \quad (2)$$

Suppose X is a non-negative random variable with probability density function $f(x)$. Let $w(x)$ be the non-negative weight function, then, the probability density function of the Weighted random variable X_w is given by

$$f_w(x)=\frac{w(x)f(x)}{E(w(x))},\quad x>0.$$

Where $w(x)$ be a non negative weight function and $E(w(x))=\int w(x)f(x)dx<\infty.$

For different weighted models, we have different choices of the weight function *w(x)*, When w(x) = x^c, then the resulting distribution is called weighted distribution. In this paper, we will obtain the length biased OM distribution. So we will take c = 1 in weights x^c, in order to get the length biased OM distribution and its pdf is given by

$$f_l(x)=\frac{xf(x)}{E(x)},\ x>0 \quad (3)$$

Where $E(x)=\int_0^\infty xf(x;\theta)dx$

$$E(x)=\frac{\theta^4+8\theta^3+36\theta^4+96\theta+120}{\theta(\theta^4+4\theta^3+12\theta^2+24\theta+24)} \quad (4)$$

On substituting the equation (1) and (4) in equation (3), we will obtain the probability density function of length biased OM distribution which is given by

$$f_l(x;\theta)=\frac{x\theta^6(1+x)^4e^{-\theta x}}{\theta^4+8\theta^3+36\theta^4+96\theta+120} \quad (5)$$

and the cumulative distribution function of length biased OM distribution is given by

$$F_l(x;\theta) = \int_0^x f_l(x;\theta)dx$$

$$F_l(x;\theta) = \int_0^x \frac{x\theta^6}{\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120}(1+x)^4 e^{-\theta x} dx$$

$$F_l(x;\theta) = \frac{1}{\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120}\int_0^x x\theta^6 (1+x)^4 e^{-\theta x} dx \qquad (6)$$

after simplification of equation (6), we will obtain the cumulative distribution function of the length biased OM distribution

$$F_l(x;\theta) = \frac{\theta^4\gamma(2,\theta x) + 4\theta^3\gamma(3,\theta x) + 6\theta^2\gamma(4,\theta x) + 4\theta\gamma(5,\theta x) + \gamma(6,\theta x)}{\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120} \qquad (7)$$

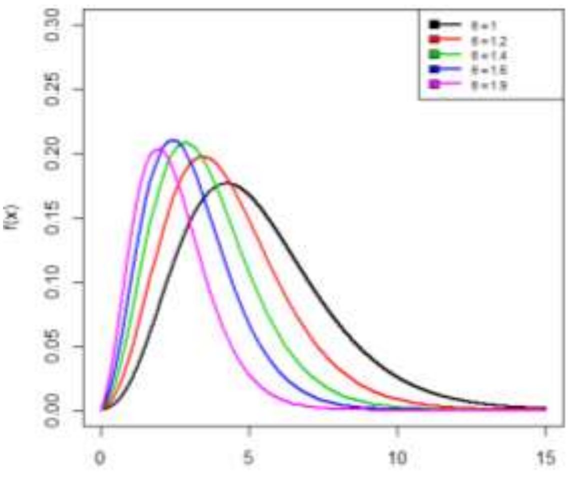


Fig 1: Pdf plot of length biased OM distribution

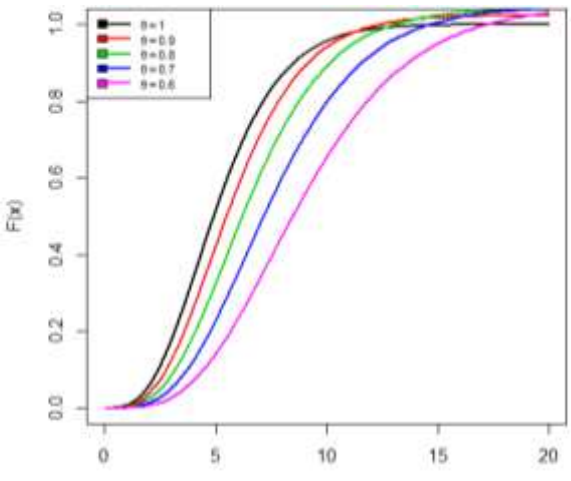


Fig 2: Cdf plot of length biased OM distribution

3. RELIABILITY ANALYSIS

In this sub-section, we will discuss the reliability function, hazard function and the reverse hazard function for the length biased OM distribution.

3.1 Reliability function

The reliability function is defined as the probability that a system survives beyond a specified time. It is also referred to as survival or survivor function of the distribution. it can be computed as complement of the cumulative distribution function of the model. The reliability function or the survival function of length biased OM distribution is calculated as

$$S(x) = 1 - F_l(x;\theta)$$

$$S(x) = 1 - \frac{\theta^4 \gamma(2,\theta x) + 4\theta^3 \gamma(3,\theta x) + 6\theta^2 \gamma(4,\theta x) + 4\theta\gamma(5,\theta x) + \gamma(6,\theta x)}{\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120}$$

3.2 Hazard function

The hazard function is also known as hazard rate or instantaneous failure rate or force of mortality and is given by

$$h(x) = \frac{f_l(x;\theta)}{R(x)}$$

$$h(x) = \frac{x\theta^6 (1+x)^4 e^{-\theta x}}{\left(\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120\right) - \left(\theta^4 \gamma(2,\theta x) + 4\theta^3 \gamma(3,\theta x) + 6\theta^4 \gamma(4,\theta x) + 4\theta\gamma(5,\theta x) + \gamma(6,\theta x)\right)}$$

3.2 Reverse hazard function

The reverse hazard function of length biased OM distribution is given by

$$h_r(x) = \frac{f_l(x;\theta)}{F_l(x;\theta)}$$

$$h_r(x) = \frac{x\theta^6}{\left(\theta^4 \gamma(2,\theta x) + 4\theta^3 \gamma(3,\theta x) + 6\theta^4 \gamma(4,\theta x) + 4\theta\gamma(5,\theta x) + \gamma(6,\theta x)\right)} (1+x)^4 e^{-\theta x}$$

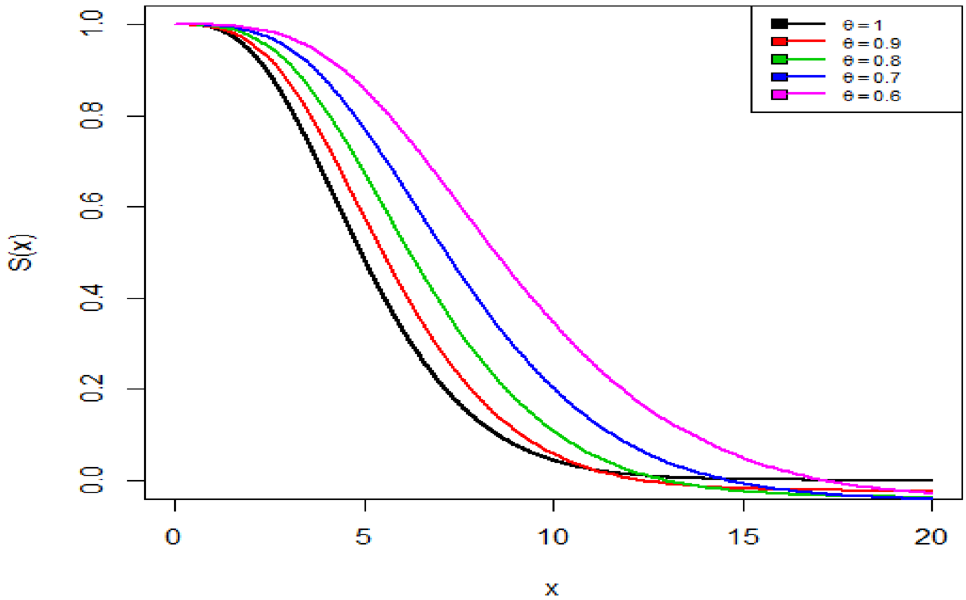


Fig.3: Survival function curves of length biased OM distribution

4. STATISTICAL PROPERTIES

In this section we shall discuss the Structural properties of length biased OM distribution.

4.1 Moments

Suppose X denotes the random variable of length biased OM distribution with parameter θ, then

$$E(X^r) = \mu_r' = \int_0^\infty x^r f_l(x;\theta)dx$$

$$E(X^r) = \int_0^\infty x^r \frac{x\theta^6}{\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120}(1+x)^4 e^{-\theta x}dx$$

$$E(X^r) = \frac{\theta^6}{\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120}\int_0^\infty x^{r+1}(1+x)^4 e^{-\theta x}dx$$

$$E(X^r) = \frac{\theta^6}{\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120}\int_0^\infty x^{r+1}(1+4x+6x^2+4x^3+x^4)e^{-\theta x}dx$$

$$E(X^r) = \frac{\theta^6}{\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120} \times \left(\begin{array}{l}\int_0^\infty x^{(r+2)-1}e^{-\theta x}dx + 4\int_0^\infty x^{(r+3)-1}e^{-\theta x}dx \\ + 6\int_0^\infty x^{(r+4)-1}e^{-\theta x}dx + 4\int_0^\infty x^{(r+5)-1}e^{-\theta x}dx + \int_0^\infty x^{(r+6)-1}e^{-\theta x}dx\end{array}\right)$$

$$E(X^r) = \mu_r' = \frac{\theta^4\Gamma(r+2) + 4\theta^3\Gamma(r+3) + 6\theta^4\Gamma(r+4) + 4\theta\Gamma(r+5) + \Gamma(r+6)}{\theta^r\left(\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120\right)} \quad (8)$$

Substitute r = 1,2,3,4 in equation (8), we get the first four moments of length biased OM distribution

$$E(X) = \mu_1' = \frac{2\theta^4 + 24\theta^3 + 144\theta^4 + 480\theta + 720}{\theta\left(\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120\right)}$$

$$E(X^2) = \mu_2' = \frac{6\theta^4 + 96\theta^3 + 720\theta^4 + 2880\theta + 5040}{\theta^2\left(\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120\right)}$$

$$E(X^3) = \mu_3' = \frac{24\theta^4 + 480\theta^3 + 4320\theta^4 + 20160\theta + 161280}{\theta^3\left(\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120\right)}$$

$$E(X^4) = \mu_4' = \frac{\theta^4\Gamma(6) + 4\theta^3\Gamma(7) + 6\theta^4\Gamma(8) + 4\theta\Gamma(9) + \Gamma(10)}{\theta^4\left(\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120\right)}$$

$$\text{Variance } (\mu_2) = \frac{\theta^4\Gamma(4) + 4\theta^3\Gamma(5) + 6\theta^4\Gamma(6) + 4\theta\Gamma(7) + \Gamma(8)}{\theta^2\left(\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120\right)} - \left(\frac{\theta^4\Gamma(3) + 4\theta^3\Gamma(4) + 6\theta^4\Gamma(5) + 4\theta\Gamma(6) + \Gamma(7)}{\theta\left(\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120\right)}\right)^2$$

$$S.D(\sigma) = \sqrt{\left(\frac{\theta^4\Gamma(4) + 4\theta^3\Gamma(5) + 6\theta^4\Gamma(6) + 4\theta\Gamma(7) + \Gamma(8)}{\theta^2\left(\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120\right)} - \left(\frac{\theta^4\Gamma(3) + 4\theta^3\Gamma(4) + 6\theta^4\Gamma(5) + 4\theta\Gamma(6) + \Gamma(7)}{\theta\left(\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120\right)}\right)^2\right)}$$

4.2 Moment Generating Function and Characteristics Function

In this sub section we derive the moment generating function and the characteristics function of length biased OM distribution. We begin with the well-known definition of the moment generating function given by

$$M_X(t) = E\left(e^{tx}\right) = \int_0^\infty e^{tx} f_l(x;\theta,)dx$$

$$M_X(t) = \int_0^\infty \left[1 + tx + \frac{(tx)^2}{2!} + \ldots.\right] f_l(x;\theta)dx$$

$$M_X(t) = \int_0^\infty \sum_{j=0}^\infty \frac{t^j}{j!} x^j f_l(x;\theta)dx$$

$$M_X(t) = \sum_{j=0}^\infty \frac{t^j}{j!} \mu_j{}'$$

$$M_X(t) = \sum_{j=0}^\infty \frac{t^j}{j!} \left[\frac{\theta^4\Gamma(j+2) + 4\theta^3\Gamma(j+3) + 6\theta^4\Gamma(j+4) + 4\theta\Gamma(j+5) + \Gamma(j+6)}{\theta^j(\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120)}\right]$$

$$M_X(t) = \frac{1}{(\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta) + 120)} \times \sum_{j=0}^\infty \frac{t^j}{j!\theta^j}\left(\theta^4\Gamma(j+2) + 4\theta^3\Gamma(j+3) + 6\theta^4\Gamma(j+4) + 4\theta\Gamma(j+5) + \Gamma(j+6)\right)$$

Similarly, the characteristics function of length biased OM distribution can be obtained as:

$$\varphi_X(t) = M_X(it)$$

$$M_X(it) = \frac{1}{(\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120)} \times \sum_{j=0}^{\infty} \frac{(it)^j}{j!\,\theta^j} \left(\theta^4 \Gamma(j+2) + 4\theta^3 \Gamma(j+3) + 6\theta^4 \Gamma(j+4) + 4\theta\Gamma(j+5) + \Gamma(j+6) \right)$$

5. LIKELIHOOD RATIO TEST

Let $X_1, X_2, X_3 \ldots\ldots X_n$ be a random sample of size n from the OM distribution or length biased OM distribution. We test the hypothesis

$$H_o : f(x) = f(x;\theta) \quad \text{against} \quad H_1 : f(x) = f_l(x;\theta)$$

For testing whether the random sample of size *n* comes from OM distribution or length biased OM distribution, the following test statistic is used

$$\Delta = \frac{L_1}{L_o} = \prod_{i=1}^{n} \frac{f_l(x;\theta)}{f(x;\theta)}$$

$$\Delta = \frac{L_1}{L_o} = \prod_{i=1}^{n} \left[\frac{\theta\, x_i (\theta^4 + 4\theta^3 + 12\theta^2 + 24\theta + 24)}{\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120} \right]$$

$$\Delta = \frac{L_1}{L_o} = \left(\frac{\theta(\theta^4 + 4\theta^3 + 12\theta^2 + 24\theta + 24)}{\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120} \right)^n \prod_{i=1}^{n} x_i$$

We reject the null hypothesis if

$$\Delta = \left(\frac{\theta(\theta^4 + 4\theta^3 + 12\theta^2 + 24\theta + 24)}{\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120} \right)^n \prod_{i=1}^{n} x_i > k$$

Equivalently, we reject the null hypothesis

$$\Delta^* = \prod_{i=1}^{n} x_i > k \left(\frac{\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120}{\theta(\theta^4 + 4\theta^3 + 12\theta^2 + 24\theta + 24)} \right)^n$$

$$\Delta^* = \prod_{i=1}^{n} x_i > k^*, \text{ Where } k^* = k\left(\frac{\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120}{\theta(\theta^4 + 4\theta^3 + 12\theta^2 + 24\theta + 24)}\right)^n$$

For a large sample of size *n*, *2log Δ* is distributed as chi-square distribution with one degree of freedom and also p- value is obtained from the chi-square distribution. Also, we reject the null hypothesis, when the probability value is given by

$p(\Delta^* > \beta^*)$, Where $\beta^* = \prod_{i=1}^{n} x_i$ is less than a specified level of significance and $\prod_{i=1}^{n} x_i$ is the observed value of the statistic Δ^*.

6. ORDER STATISTICS

Order statistics make their appearance in many statistical theory and practice. We know that if $X_{(1)}$, $X_{(2)}$, ..., $X_{(n)}$ denotes the order statistics of a random sample $X_1, X_2, \ldots, X_n$ from a continuous population with cdf $F_X(x)$ and pdf $f_x(x)$, then the pdf of *r*th order statistics $X_{(r)}$ is given by

$$f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!} f_X(x)\left(F_X(x)\right)^{r-1}\left(1 - F_X(x)\right)^{n-r} \tag{9}$$

Using equation (5) and (7) in equation (9) the expression of the *rth* order statistics X_r of length biased OM distribution is given by

$$f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!}\left(\frac{x\theta^6}{\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120}(1+x)^4 e^{-\theta x}\right)$$
$$\times\left(\frac{1}{\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120}\left(\begin{array}{l}\theta^4\gamma(2,\theta x) + 4\theta^3\gamma(3,\theta x)\\ + 6\theta^2\gamma(4,\theta x) + 4\theta\gamma(5,\theta x) + \gamma(6,\theta x)\end{array}\right)\right)^{r-1}$$
$$\times\left(1 - \frac{1}{\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120}\left(\begin{array}{l}\theta^4\gamma(2,\theta x) + 4\theta^3\gamma(3,\theta x)\\ + 6\theta^2\gamma(4,\theta x) + 4\theta\gamma(5,\theta x) + \gamma(6,\theta x)\end{array}\right)\right)^{n-r}$$

Therefore, the expression of the higher order statistics $X_{(n)}$ of length biased OM distribution is given by

$$f_{x(n)}(x) = \frac{nx\theta^6}{\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120}(1+x)^4 e^{-\theta x}$$
$$\times\left(\frac{1}{\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120}\begin{pmatrix}\theta^4\gamma(2,\theta x) + 4\theta^3\gamma(3,\theta x) \\ +6\theta^2\gamma(4,\theta x) + 4\theta\gamma(5,\theta x) + \gamma(6,\theta x)\end{pmatrix}\right)^{n-1}$$

and the expression of the first order statistics $X_{(1)}$ of length biased OM distribution is given by

$$f_{x(1)}(x) = \frac{nx\theta^6}{\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120}(1+x)^4 e^{-\theta x}$$
$$\times\left(1 - \frac{1}{\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120}\begin{pmatrix}\theta^4\gamma(2,\theta x) + 4\theta^3\gamma(3,\theta x) \\ +6\theta^2\gamma(4,\theta x) + 4\theta\gamma(5,\theta x) + \gamma(6,\theta x)\end{pmatrix}\right)^{n-1}$$

7. ENTROPIES

The concept of entropies is important in different areas such as probability and statistics, physics, communication theory and economics. Entropies quantify the diversity, uncertainty, or randomness of a system. Entropy of a random variable X is a measure of variation of the uncertainty.

7.1 Renyi Entropy

The Renyi entropy is important in ecology and statistics as index of diversity. The Renyi entropy is also important in quantum information, where it can be used as a measure of entanglement. For a given probability distribution, Renyi entropy is given by

$$e(\beta) = \frac{1}{1-\beta}\log\left(\int f_l^{\beta}(x)dx\right)$$

Where, $\beta > 0$ and $\beta \neq 1$

$$e(\beta) = \frac{1}{1-\beta}\log\int_0^{\infty}\left(\frac{x\theta^6}{\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120}(1+x)^4 e^{-\theta x}\right)^{\beta} dx$$

$$e(\beta) = \frac{1}{1-\beta}\log\left(\left(\frac{\theta^6}{\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120}\right)^{\beta}\int_0^{\infty} x^{\beta} e^{-\theta\beta x}(1+x)^{4\beta} dx\right)$$

Using Binomial expansion, we obtain

$$e(\beta) = \frac{1}{1-\beta}\log\left(\left(\frac{\theta^6}{\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120}\right)^{\beta}\sum_{j=0}^{\infty}\binom{\beta}{j}1^{4\beta-j}x^j\int_0^{\infty} x^{\beta} e^{-\theta\beta x}dx\right)$$

$$e(\beta)=\frac{1}{1-\beta}\log\left(\left(\frac{\theta^6}{\theta^4+8\theta^3+36\theta^4+96\theta+120}\right)^{\beta}\sum_{j=0}^{\infty}\binom{\beta}{j}\int_0^{\infty}x^{(\beta+j+1)-1}e^{-\theta\beta x}dx\right)$$

$$e(\beta)=\frac{1}{1-\beta}\log\left(\left(\frac{\theta^6}{\theta^4+8\theta^3+36\theta^4+96\theta+120}\right)^{\beta}\sum_{j=0}^{\infty}\binom{\beta}{j}\frac{\Gamma(\beta+j+1)}{(\theta\beta)^{\beta+j+1}}\right)$$

7.2 Tsallis Entropy

A generalization of Boltzmann-Gibbs (B.G) statistical properties initiated by Tsallis has focused a great deal to attention. This generalization of B-G statistics was proposed firstly by introducing the mathematical expression of Tsallis entropy (Tsallis, 1988) for a continuous random variable is defined as follows

$$S_{\lambda}=\frac{1}{\lambda-1}\left(1-\int_0^{\infty}f_W{}^{\lambda}(x)dx\right)$$

$$S_{\lambda}=\frac{1}{\lambda-1}\left(1-\int_0^{\infty}\left(\frac{x\theta^6}{\theta^4+8\theta^3+36\theta^4+96\theta+120}(1+x)^4e^{-\theta x}\right)^{\lambda}dx\right)$$

$$S_{\alpha}=\frac{1}{\lambda-1}\left(1-\left(\frac{\theta^6}{\theta^4+8\theta^3+36\theta^4+96\theta+120}\right)^{\lambda}\int_0^{\infty}x^{\lambda}e^{-\lambda\theta x}(1+x)^{4\lambda}dx\right) \quad (10)$$

Using Binomial expansion in equation (10), we get

$$S_{\lambda}=\frac{1}{\lambda-1}\left(1-\left(\frac{\theta^6}{\theta^4+8\theta^3+36\theta^4+96\theta+120}\right)^{\lambda}\sum_{j=0}^{\infty}\binom{\lambda}{j}1^{4\lambda-j}x^j\int_0^{\infty}x^{\lambda}e^{-\lambda\theta x}dx\right)$$

$$S_{\lambda}=\frac{1}{\lambda-1}\left(1-\left(\frac{\theta^6}{\theta^4+8\theta^3+36\theta^4+96\theta+120}\right)^{\lambda}\sum_{j=0}^{\infty}\binom{\lambda}{j}\int_0^{\infty}x^{(\lambda+j+1)-1}e^{-\lambda\theta x}dx\right)$$

$$S_{\lambda}=\frac{1}{\lambda-1}\left(1-\left(\frac{\theta^6}{\theta^4+8\theta^3+36\theta^4+96\theta+120}\right)^{\lambda}\sum_{j=0}^{\infty}\binom{\lambda}{j}\frac{\Gamma(\lambda+j+1)}{(\lambda\theta)^{\lambda+j+1}}\right)$$

8. BONFERRONI AND LORENZ CURVES

The Bonferroni and the Lorenz curves are used not only in economics to study income and poverty, but it is also being used in other fields like reliability, medicine, insurance and demography. The Bonferroni and Lorenz curves are given by

$$B(p) = \frac{1}{p\mu_1'}\int_0^q x\, f_l(x;\theta)dx$$

and $L(p) = pB(p) = \frac{1}{\mu_1'}\int_0^q x\, f_l(x;\theta)dx$

Where $\mu_1' = E(X) = \dfrac{\theta^4\Gamma(3)+4\theta^3\Gamma(4)+6\theta^4\Gamma(5)+4\theta\Gamma(6)+\Gamma(7)}{\theta(\theta^4+8\theta^3+36\theta^4+96\theta+120)}$ and $q = F^{-1}(p)$

$$B(p) = \frac{\theta(\theta^4+8\theta^3+36\theta^4+96\theta+120)}{p\left(\theta^4\Gamma(3)+4\theta^3\Gamma(4)+6\theta^4\Gamma(5)+4\theta\Gamma(6)+\Gamma(7)\right)}\times\int_0^q \frac{\theta^6}{\theta^4+8\theta^3+36\theta^4+96\theta+120}x^2(1+x)^4e^{-\theta x}dx$$

$$B(p) = \frac{\theta^7}{p\left(\theta^4\Gamma(3)+4\theta^3\Gamma(4)+6\theta^4\Gamma(5)+4\theta\Gamma(6)+\Gamma(7)\right)}\int_0^q x^2(1+4x+6x^2+4x^3+x^4)e^{-\theta x}dx$$

$$B(p) = \frac{\theta^7}{p\left(\theta^4\Gamma(3)+4\theta^3\Gamma(4)+6\theta^4\Gamma(5)+4\theta\Gamma(6)+\Gamma(7)\right)}$$

$$\times\left(\int_0^q x^{(3)-1}e^{-\theta x}dx+4\int_0^q x^{(4)-1}e^{-\theta x}dx+6\int_0^q x^{(5)-1}e^{-\theta x}dx+4\int_0^q x^{(6)-1}e^{-\theta x}dx+\int_0^q x^{(7)-1}e^{-\theta x}dx\right)$$

$$B(p) = \frac{\theta^7\left(\gamma(3,\theta q)+4\gamma(4,\theta q)+6\gamma(5,\theta q)+4\gamma(6,\theta q)+\gamma(7,\theta q)\right)}{p\left(\theta^4\Gamma(3)+4\theta^3\Gamma(4)+6\theta^4\Gamma(5)+4\theta\Gamma(6)+\Gamma(7)\right)}$$

and $L(p) = pB(p) = \dfrac{\theta^7\left(\gamma(3,\theta q)+4\gamma(4,\theta q)+6\gamma(5,\theta q)+4\gamma(6,\theta q)+\gamma(7,\theta q)\right)}{\left(\theta^4\Gamma(3)+4\theta^3\Gamma(4)+6\theta^4\Gamma(5)+4\theta\Gamma(6)+\Gamma(7)\right)}$

9. PARAMETER ESTIMATION AND FISHER'S INFORMATION MATRIX

In this section, we will discuss the parameter estimation of length biased OM distribution using maximum likelihood method. Let $x_1, x_2, x_3, \ldots. x_n$ be a random sample of size n from the length biased OM distribution and let f_x be the observed frequency in the sample corresponding to $X = x(x = 1,2,\ldots.,$

n) such that $\sum_{x=1}^{n} f_x = n,$ Where n is the largest observed value having non-zero frequency. Then the likelihood function of length biased OM distribution is given by

$$L(x;\theta) = \prod_{i=1}^{n} f_w(x;\theta)$$

$$L(x;\theta) = \prod_{i=1}^{n} \left[\frac{x_i \theta^6}{\left(\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120\right)} (1+x_i)^4 e^{-\theta x_i} \right]$$

$$L(x;\theta) = \frac{\theta^{6n}}{\left(\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120\right)^n} \prod_{i=1}^{n} \left[x_i (1+x_i)^4 e^{-\theta x_i} \right]$$

The log likelihood function is given by

$$\log L(x;\theta) = 6n \log\theta - n\log\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120 + \sum_{i=1}^{n} \log x_i + 4\sum_{i=1}^{n} \log(1+x_i) - \theta\sum_{i=1}^{n} x_i \quad (11)$$

Differentiating the equation (11) partially w.r.to θ and equating to zero, we will have the following system of equations

$$\frac{\partial \log L}{\partial\theta} = \frac{6n}{\theta} - n\left(\frac{4\theta^3 + 12\theta^2\Gamma(3) + 24\theta^3\Gamma(4) + 4\Gamma(5)}{\theta^4 + 8\theta^3 + 36\theta^4 + 96\theta + 120} \right) - \sum_{i=1}^{n} x_i = 0 \quad (12)$$

Because of the complicated form of the likelihood equation, algebraically it is very difficult to solve the system of non-linear equation. Therefore we use R and wolfram mathematica for estimating the required parameter.

10. APPLICATIONS

In this section, we have used a real-life data set to show that the length biased OM distribution can be a better model than the OM distribution. The data set is given below

Data Set 1: The data set given in table 1 represents the non-censored data represents the survival times (in months) of 32 patients of melanoma (non-censored data) has been taken from Kayid et.al . (2010).

Table 1: Survival times (in months) of 32 patients of melanoma (non-censored data)

3.25	3.5	4.75	4.75	5	5.25	5.75	5.75
6.25	6.5	6.5	6.75	6.75	7.78	8	8.5
8.5	9.25	9.5	9.5	10	11.5	12.5	13.25
13.5	14.25	14.5	14.75	15	16.25	16.25	16.5

In order to compare the performance of length biased OM distribution with OM distribution. We are using the Criterion values like AIC (Akaike information criterion), AICC (corrected Akaike information criterion) and BIC (Bayesian information criterion). The better distribution corresponds to lesser AIC, AICC and BIC values. The formulae for calculation of AIC, BIC and AICC are

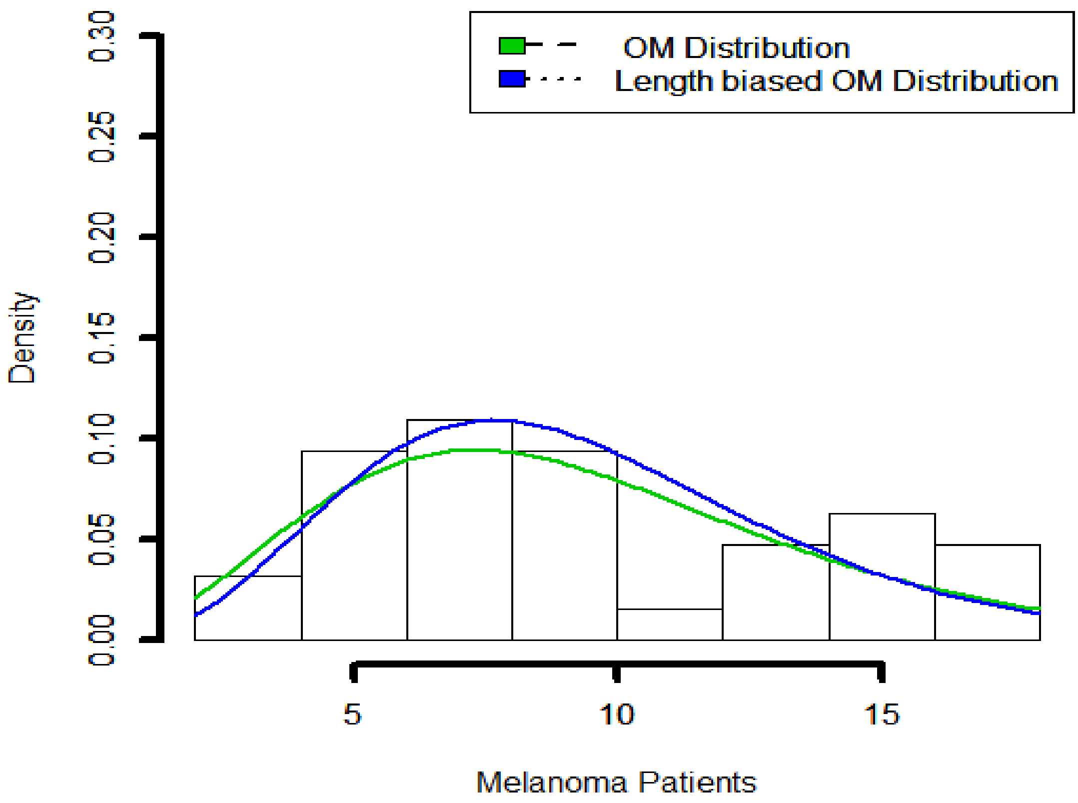
$$\text{AIC} = 2k - 2\log L \qquad \text{AICC} = AIC + \frac{2k(k+1)}{n-k-1} \quad \text{and} \quad \text{BIC} = k\log n - 2\log L$$

Where k is the number of parameters, n is the sample size and -2logL is the maximized value of the log-likelihood function under the considered model. Here we are using R-software to find the above values. From table 1, it has been observed that the length biased OM distribution have the lesser AIC, AICC, -2log$L$ and BIC values as compared to OM distribution. Hence we can conclude that the length biased OM distribution leads to a better fit than the OM distribution.

Table 2- Parameter Estimates, AIC, BIC, AICC, -2logL criterion values of the fitted distribution

Data set	Distribution	MLE	S.E.	-2logL	AIC	BIC	AICC
1	**OM**	$\hat{\theta} = 0.48194936$	$\hat{\theta} = 0.03811025$	178.5129	180.5129	181.9786	181.37
	Length biased OM	$\hat{\theta} = 0.59319710$	$\hat{\theta} = 0.04224115$	174.6473	176.6473	178.113	177.5044

Fig.4: Fitting density curves of Data set



11. CONCLUSION

In this paper, we have obtained the length biased version of OM distribution. The subject distribution is generated by using the length biased technique. The moments, harmonic mean, survival function, hazard function and the maximum likelihood estimation of the parameters of the distribution have also been obtained. The application of the new distribution has also been demonstrated with a real life data set. The results are compared with OM distribution and the results indicate that the length biased OM distribution provides a better fit than the OM distribution.

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