EFFECTIVE NUMERICAL METHOD FOR CONSTRUCTING BAYESIAN CHAIN SAMPLING PLAN (BChSP-1)

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Abstract

This paper deals with the collection of tables for the selection of the Bayesian Chain Sampling Plan (BChSP-1), which introduces a new procedure for BChSP-1 indexed by Acceptable and Limiting Quality Levels with the application of Numerical Method including Linear Interpolation Method, Secant Method, Muller Method, Modified Muller Method. Tables are provide the paln parameter values.

Keywords: Bayesian Chain Sampling Plan, Linear Interpolation Method, Secant Method, Muller's Method, Modified Muller's Method, Acceptable and Limiting Quality Levels.

I. INTRODUCTION

The use of sample information is geared towards Bayesian Statistics. In inductive inference, Thomas Bayes (1702-1761) was the first to use prior knowledge and the approach to statistics, which formally attempts to use prior information, is called Bayesian analysis.If a product in a series supplies a product, these lots will vary in quality due to random variations, even though the process is stable and in control. In the lot (sampling) variations of individual units and between lot (sampling and process) variations, these fluctuations can be segregated.

The Bayesian Acceptance Sampling Method is related to the use of prior distribution selection process history (e.g., Gamma Poisson, Beta Binomial) to explain the random fluctuations in Acceptance Sampling. The consumer is allowed by Bayesian sampling plans to specifically define the distribution of defectives from lot to lot.The prior distribution is the anticipated distribution of a lot of quality that the sampling strategy would run on. The distribution is named earlier since it is formulated before samples are taken. The combination of prior knowledge, defined by the previous distribution, and empirical knowledge based on the sample leads to the decision on the lot.

Cumulative-results sampling inspection plans generally use the current as well as past sample information from product entities inmaking a decision about the current product entities. [See, Stephens (1966)]. A class of cumulative-results plans has been developed based on the concept and the procedure introduced by Dodge (1955). The basic procedure is labeled as chain sampling plan and is designated as ChSP-1. An extension of ChSP-1 is the dependent and deferred state sampling plans of Wortham and Mogg (1971), and Wortham and Baker (1976). The cumulative results plans, including ChSP-1, are applied under the conditions that (a) the production is reasonably steady so that the results on current and preceding lots are broadly indicative of a continuing process; (b) samples from lots are obtained essentially in the order of production; (c) inspection is by attributes with quality defined in terms of a fraction defective, , p and (d) lots are expected to be essentially of same quality. However, it is rather unrealistic to often expect a production process to produce lots of same or similar quality as the lots will have quality variations, which occur due to random fluctuations of the process. In such cases, Bayesian methodology under sampling plans could be employed with the appropriate choice of prior distributions about the process parameters.

The chain sampling plan (ChSP-1) proposed by Dodge (1955), making use of cumulative results of several samples, helps to overcome these short comings of single sampling c=0 plans. It avoids rejection of a lot on the basis of a single non conforming unit and improves the poor discrimination between good and bad quality that occurs with the c=0 plan. Suresh and Pradeepa veerakumari (2007) introduce some procedures and tables for the selection of for Bayesian Chain Sampling Plan. Suresh and Sangeetha (2011) developed the Bayesian Chain Sampling Plan using (BChSP-1) Quality Region.

II. BAYESIAN CHAIN SAMPLING PLAN (BChSP-1)

Thus according Dodge (1955), the operational characteristic feature of ChSP-1 is

$$P_a(p) = P_0 + P_1 (P_0)^i$$

Two parameters n and i define the Chain Sampling Plan (ChSP-1), where n is the sample size and i is the number of previous samples with zero defects, using the OC curve, Dodge (1955) has studied the Chain Sampling Plan properties. Additional OC curves, which cover most of the conditions, have been presented by Clark (1960).Soundararajan (1978 a, b) has defined procedures and tables indexed by stated parameters for the construction and selection of chain sampling plans (ChSP-1). The likelihood of ChSP-1 acceptance based on the Poisson model is given as

$$P(n,i/p) = e^{-np} + e^{-np(1+i)} np$$

Using the previous experience of inspection, it is noted that p fits the Gamma distribution with density function,

$$w(p) = e^{-pt} p^{s-i} t^s / \Phi(s), \qquad s, t > 0, p > 0$$

The average probability of acceptance is given as

$$\overline{p}=\int_0^\infty P(n,i\ /\ p)w(\ p)dp$$

$$\overline{p}= s^{S}/(s+n\mu)^{S}+ns^{S+1}\mu/(s+n\mu+ni\mu)^{S+1}$$

Where $\mu=s\ /\ t$, is the mean value of the product quality p.

Differentiating the APA function with respect to μ gives

$$\frac{d\overline{p}}{d\mu}=\frac{-ns^{s+1}}{(s+n\mu)^{s+1}}+\frac{ns^{s+2}(1-n\mu-n\mu i)}{(s+n\mu+n\mu i)^{s+2}}$$

The relative slope h at μ is,

$$h=\frac{-\mu}{\overline{p}(\mu)}\frac{d\overline{p(\mu)}}{d\mu}$$

Differentiating the APA function with respect to μ and evaluating at μ we get various values of (i, s).

III. USING FOUR NUMERICAL ITERATIVE PROCEDURE FOR THE DEVELOPMENT OF PERFORMANCE MEASURES

a. LINEAR INTERPOLATION METHOD: Easy to Convergent also calculate the error.

b. SECANT METHOD: Extension and Alternative to Newton-Raphson method.

c. MULLER'S METHOD: Extension of Secant method.

d. MODIFIED MULLER'S METHOD (MMM): Extension of Muller's Method.

3.1. Linear Interpolation Method

Step 1: Select two initial values x_1 and x_2 where $f(x_1)$ and $f(x_2)$ are of opposite signs.
Step 2: Set $f_1= f\ (x_1)$ and $f_{2=}\ f\ (x_2)$.
Step 3: Set $x_3 = x_2 - \frac{f_2(x_2-x_1)}{f_2-f_1}$
Step 4: Calculate $f\ (x_3)$ and set $f_3 = f\ (x_3)$.
Step 5:

a) if f_3 =0 or if the desired precision is met x_3 is taken as the required root value. Then go to step 7.
b) if f_3 is of an opposite is not met, then go to step 6.

Step 6:
a) if f_3 is of the opposite sign of f_1, then set x_2=x_3.

b) if f_3 is of the same sign as that of f_1, then reset $x_1 = x_3$. Go to step 2.
Step 7: Stop.

3.2. Secant Method

Step 1: Select two initial values x_1 and x_2 where $f(x_1)$ and $f(x_2)$ are of opposite signs.
Step 2: Set $f_1 = f(x_1)$ and $f_2 = f(x_2)$.
Step 3: Set
$$x_3 = x_2 + \frac{f_2(x_2 - x_1)}{f_1 - f_2}$$
Step 4: Calculate $f(x_3)$ and set $f_3 = f(x_3)$.
Step 5:
a) if $f_3 = 0$ or if the desired precision is met x_3 is taken as the required root value. Then go to step 7.
b) if $f_3 \neq 0$ and if the desired precision is not met then go to step 6.
Step 6:
a) if f_3 is not met, then reset $x_2 = x_3$.
b) if f_3 is of the same sign as that of f_1, then reset $x_1 = x_3$. Go to step 2.
Step 7: Stop.

3.3. Muller's Method

Step 1: Start with three initial values x_2, x_0, and x_1.
Step 2: calculate $f(x_2)$, $f(x_0)$ and $f(x_1)$, Set $f_1 = f(x_1)$, $f_0 = f(x_0)$ and $f_2 = f(x_2)$. Calculate $f(x_0)$ and set $f_0 = f(x_0)$, $c = f_0$
Step 3: Calculate $h_1 = x_1 - x_0$ and $h_2 = x_0 - x_2$, $\lambda = \frac{h2}{h1}$,
Step 4: $a = \frac{\lambda f_1 - f_0(1+\lambda) + f_2}{\lambda {h_1}^2 (1+\lambda)}$

$b = \frac{f_1 - f_0 - a{h_1}^2}{h_1}$

Step 5: calculate the value of the root
$$x = x_0 - \frac{2a}{b + \sqrt{b^2 - 4ac}} \qquad \text{if } b<0$$
$$x = x_0 - \frac{2a}{b - \sqrt{b^2 - 4ac}} \qquad \text{if } b>0$$
$$x = x_0 - \left(\pm\sqrt{-\frac{c}{a}}\right) \qquad \text{if } b=0$$
Step 6:
a) If $f(x) = 0$ or if the desired precision is met, x is taken as the required root value. Then go to Step 8.
b) if $f(x) \neq 0$ and the desired precision are not met, then go to step 7.
Step 7:
a) if x>x0, then reset x2=x0, x0= x, x1=x1.
b) if $x < x0$, then reset $x_2 = x_2$, $x_0 = x$, $x_1 = x_0$.
Go to step 2.
Step 8: Stop

3.4. Modified Muller Method

Step 1:, Set $f_1 = f(x_1)$, $f_2 = f(x_2)$.
Step 2: set x
$x_0 = x_1 - \frac{f_1(x_1 - x_2)}{f_1 - f_2}$
Step 3: Calculate f (x_0) and set $f_0 = f(x_0)$, $c = f_0$
Step 4: Calculate $h_1 = x_1 - x_0$ and $h_2 = x_0 - x_2$, $\lambda = \frac{h2}{h1}$,
Step 5: $a = \frac{\lambda f_1 - f_0(1+\lambda) + f_2}{\lambda {h_1}^2 (1+\lambda)}$

$b = \frac{f_1 - f_0 - a{h_1}^2}{h_1}$

Step 6: calculate the value of the root $x = x_0 - \frac{2a}{b + \sqrt{b^2 - 4ac}}$ if b>0

$$x = x_0 - \frac{2a}{b - \sqrt{b^2 - 4ac}} \text{ if } b<0$$

$$x = x_0 - \left(\pm\sqrt{-\frac{c}{a}}\right) \text{ if } b=0$$

Step 7:
a) If f(x)=0 or if the desired precision is met, x is taken as the required root value. Then go to Step 9.
b) if f(x)≠0 and the desired precision are not met, then go to step 8.
Step 8:
a) if x>x0, then reset x2=x0, x0= x, x1=x1.
b) if x<x0, then reset $x_2 = x_2$, $x_0 = x$, $x_1 = x_0$.
Go to step 3.
Step 9: Stop

IV. DESIGNING PLANS FOR GIVEN AQL, LQL, Α AND Β

1. Specify *p1* = Acceptable Quality Level at α = 0.05 or 0.01.

2. Specify *p2* = Limiting Quality Level at β = 0.10 or 0.05.

3. Obtain the corresponding ratio OR = *p2 / p1* at different combination of α and β.

4. The actual *np1* and *np2* values corresponding to the OR value has been noted.

5. Determine the sample size *n= np1 / p1*. Round up the value for determining the sample size.

6. Thus the plan consists of the parameter *n, i,* f_1, f_2, f_3 and *c*.

Selection of Parameters for Bayesian Chain Sampling Plan

Table 2 is used to select the parameters for the Bayesian Chain Sampling Plan(BChSP-1) indexed by μ and h. For example, for given μ = 0.01, h=0.9 and Pa(p)=0.95, The values of associated are nμ= 0.0917, s = 1 and i = 6. From this one can obtain the sample size as n= nμ/ μ = 9. Thus the parameters are n= 9, s = 1 and i = 6.The amount of iteration needed by all the techniques is equivalent. Any of the methods given can therefore be used for the calculation of parameters.

V. CONCLUSION

Bayesian Chain Sampling Plan Structure (BChSP-1) indexed using Linear Interpolation Method, Secant Method, Muller's Method, Modified Muller's Method to obtain the proportion defectives (nμ) for (BChSP-1) in this paper through Acceptable and Limiting Quality Levels. The work discussed in this paper relates to the current Bayesian Chain Sampling Plan(BChSP-1) construction and selection method.The findings presented in this paper are primarily related to the proposal of new procedures and required tables for the ready-to-use selection of the sampling method by means of quality levels involving producers and consumers. Tables are given that are tailor-made, functional, and ready-made for the industrial shop-floor conditions. These tables are useful for acquiring high-quality goods with lower inspection costs for both producers and customers.

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Table.1: Parametric values for Bayesian Chain Sampling Plan (BChSP-1) with comparison of various Numerical Methods:

		Linear Interpolation Method		Secant Method		Muller's Method		Modified Muller's Method	
s	i	iteration number	$\bar{P}$=0.99	iteration number	$\bar{P}$=0.99	iteration number	$\bar{P}$=0.99	iteration number	$\bar{P}$=0.99
1	0	5	0.1111	6	0.1111	5	0.1111	5	0.1111
	1	5	0.0645	6	0.0645	6	0.0645	5	0.0645
	2	6	0.0505	6	0.0505	6	0.0505	5	0.0505
	3	6	0.0432	5	0.0432	5	0.0432	6	0.0432
	4	7	0.0385	6	0.0385	5	0.0385	5	0.0385
	5	6	0.0352	5	0.0352	5	0.0352	5	0.0352
	6	7	0.0327	6	0.0327	5	0.0327	5	0.0327
3	0	7	0.1315	7	0.1315	5	0.1315	5	0.1315
	1	7	0.0763	6	0.0763	4	0.0763	5	0.0763
	2	7	0.0595	7	0.0595	5	0.0595	5	0.0595
	3	7	0.0507	7	0.0507	5	0.0507	5	0.0507
	4	6	0.045	6	0.045	4	0.045	5	0.045
	5	7	0.0416	7	0.0416	5	0.0416	5	0.0416
	6	6	0.038	6	0.038	4	0.038	5	0.038
5	0	6	0.1375	7	0.1375	5	0.1375	4	0.1375
	1	7	0.0797	7	0.0797	5	0.0797	5	0.0797
	2	6	0.0621	6	0.0621	5	0.0621	4	0.0621
	3	7	0.0529	7	0.0529	5	0.0529	4	0.0529
	4	7	0.0469	6	0.0469	4	0.0469	4	0.0469
	5	7	0.0427	7	0.0427	5	0.0427	4	0.0427
	6	7	0.0395	6	0.0395	4	0.0395	4	0.0395
7	0	6	0.1404	6	0.1404	4	0.1404	5	0.1404
	1	7	0.0813	7	0.0813	4	0.0813	4	0.0813
	2	7	0.0634	7	0.0634	4	0.0634	4	0.0634
	3	7	0.0539	7	0.0539	5	0.0539	5	0.0539
	4	7	0.0479	6	0.0479	4	0.0479	5	0.0435
	3	7	0.0435	7	0.0435	5	0.0435	5	0.0435
	4	7	0.0403	6	0.0403	4	0.0403	5	0.0403

Table.2: Parametric values for Bayesian Chain Sampling Plan (BChSP-1) with comparison of various Numerical Methods:

		Linear Interpolation Method		Secant Method		Muller's Method		Modified Muller's Method	
s	i	iteration number	$\overline{P}$=0.95	iteration number	$\overline{P}$=0.95	iteration number	$\overline{P}$=0.95	iteration number	$\overline{P}$=0.95
1	0	9	0.2880	8	0.2880	7	0.2880	6	0.2880
	1	8	0.1686	7	0.1686	6	0.1686	7	0.1686
	2	7	0.1340	6	0.1340	6	0.1340	6	0.1340
	3	8	0.1163	8	0.1163	7	0.1163	6	0.1163
	4	7	0.1052	6	0.1052	6	0.1052	6	0.1052
	5	7	0.0975	7	0.0975	7	0.0975	7	0.0975
	6	7	0.0917	8	0.0917	6	0.0917	6	0.0917
3	0	8	0.3245	6	0.3245	6	0.3245	5	0.3245
	1	7	0.1892	7	0.1892	5	0.1892	5	0.1892
	2	7	0.1493	7	0.1493	5	0.1493	6	0.1493
	3	6	0.1286	7	0.1286	6	0.1286	5	0.1286
	4	7	0.1155	6	0.1155	5	0.1155	5	0.1155
	5	6	0.1063	7	0.1063	6	0.1063	6	0.1063
	6	7	0.0994	7	0.0994	6	0.0994	6	0.0994
5	0	7	0.3353	8	0.3353	7	0.3353	7	0.3353
	1	7	0.1953	8	0.1953	6	0.1953	6	0.1953
	2	6	0.1538	7	0.1538	7	0.1538	5	0.1538
	3	7	0.1322	8	0.1322	5	0.1322	5	0.1322
	4	7	0.1185	8	0.1185	6	0.1185	6	0.1185
	5	7	0.1089	6	0.1089	5	0.1089	6	0.1089
	6	6	0.1017	8	0.1017	6	0.1017	6	0.1017
7	0	9	0.2880	9	0.2880	7	0.2880	5	0.2880
	1	8	0.1686	8	0.1686	7	0.1686	6	0.1686
	2	8	0.1340	8	0.1340	7	0.1340	5	0.1340
	3	7	0.1163	7	0.1163	7	0.1163	5	0.1163
	4	8	0.1052	7	0.1052	7	0.1052	5	0.1052
	5	8	0.0975	6	0.0975	7	0.0975	6	0.0975
	6	7	0.0917	7	0.0917	7	0.0917	6	0.0917