CONTROL CHART FOR AVERAGE FRACTION DEFECTIVES USING FUZZYFICATION

[1]S.Prabu and [2]Dr.C.Nanthakumar
[1]Ph.D Research Scholar, Department of Statistics, Salem Sowdeswari College, Salem – 636010.
[2]Assoicate Professor & Head, Department of Statistics, Salem Sowdeswari College, Salem – 636010.

Abstract

Fuzzy sets theory is powerful mathematical approach to analyse uncertainty, ambiguous and incomplete that can linguistically define data in these situations. In statistical process control, the control charts are used for products (that are currently being produced) in order to control process and prevent the production of defective components. Fuzzy control charts have been extended by converting the fuzzy sets associated with linguistic or uncertain values into scalars regarded as representative values. In this paper, we develop a new fuzzy control chart for average fraction defectives (u) using process capability with an example.

Keywords: Fuzzy, Fuzzy control chart and Process capability.

1. Introduction

Control charts were designed to monitor a process and detect shifts in mean and variance of quality characteristics to assure that the processes are performing in an acceptable manner. Two main types of control charts include variable and attribute control charts. The first is used to monitor measurable characteristics on numerical scales (Sogandi et al. 2014). Quality characteristics cannot be easily represented in numerical form monitored by second. In contrast to variable control charts, attribute control charts could monitor more than one quality characteristic simultaneously and need less cost and time for inspections. However the observation of these control charts accompany with ambiguous and vague. In classical control charts for fraction defectives, products are clearly categorized as conformed and non-conformed. In many situations, binary classification may not be appropriate since they have several intermediate levels and the necessity to apply mathematical powerful tool in order to increase the performance of control charts (Montgomery, 2000). Hence, recently fuzzy control charts have been extended to analyse uncertainty, ambiguous and incomplete or linguistically defined data. Fuzzy sets convert associated linguistic or uncertain values into scalars regarded as representative values. Gulbay and Kahraman (2004) constructed α-level fuzzy control charts for attributes data to represent the ambiguous of the data and strange of the inspection. In real-applications, there are many cases with uncertainty, ambiguous and incomplete or linguistically defined data. Obviously, mentioned data effect on the performance of attribute control chart. Hence, it is necessary to use a new approach that increases flexibility in range of observation whereas improve the performance of attribute control chart in detection of assignable cause. This research paper is summarized as the theoretical structure of fuzzy rule with control chart for average fraction defectives (u) is given below with an illustration.

2. Fuzzy triangular numbers

Fuzzy set theory is very helpful for dealing with the kind of vagueness of human thought and languagefound in a Statistical process control. In this study, a number of nonconformities will be expressedusing triangular fuzzy numbers (TFN). Let 'U'be the universe of discourse, U=[0,u].The triangularfuzzy number is defined as $\tilde{A}=(\alpha_m,\alpha_l,\alpha_r)$ also is formulated;

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & ; x \le \alpha_m - \alpha_l \\ 1 + \dfrac{x-\alpha_m}{\alpha_l} & ; \alpha_m - \alpha_1 \le x \le \alpha_m \\ 1 - \dfrac{x-\alpha_m}{\alpha_r} & ; \alpha_m \le x \le \alpha_m + \alpha_r \\ 0 & ; x \ge \alpha_m + \alpha_r \end{cases}$$

where α_m is the centre (mode); α_l is left spread; α_r is right spread.

3. Methods and materials

In the existing Shewhart (1931) method, the formulation of the control chart for average fraction defectives (u) is calculated by the following equation:

$$UCL_u = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}}$$
$$CL_u = \bar{u}$$
$$LCL_u = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}}$$

where UCL is the upper control limit, CL is the centre line and LCL is the lower control limit of ‘u’ control chart.

$$\bar{u} = \frac{\sum_{j=1}^{n} u_j}{n}$$

Fuzzy numbers $\left(b_x, b_y, b_z\right)$ are represented as $\left(\tilde{\bar{c}}_{b_{x_j}}, \tilde{\bar{c}}_{b_{y_j}}, \tilde{\bar{c}}_{b_{z_j}}\right)$ for each fuzzy observation on the control chart for average fraction defectives. The centre line (CL) for the $\tilde{u}$-control chart is as follows:

$$\tilde{\bar{u}}_{b_{x_j}} = \frac{\sum_{j=1}^{n} \tilde{u}_{b_{x\,j}}}{n}, \tilde{\bar{u}}_{b_{y_j}} = \frac{\sum_{j=1}^{n} \tilde{u}_{b_{y\,j}}}{n} \text{ and } \tilde{\bar{u}}_{b_{z_j}} = \frac{\sum_{j=1}^{n} \tilde{u}_{b_{z\,j}}}{n}$$

By reasoning about the formulations of $\tilde{u}$-control limits and fuzzy numbers primarily based on triangular membership functions, the fuzzy centre line, fuzzy upper and fuzzy lower limits of the fuzzy rule $\tilde{u}$-control chart are given as follows:

$$\left(UCL_{\tilde{u}_{b_x}}, UCL_{\tilde{u}_{b_y}}, UCL_{\tilde{u}_{b_z}}\right) = \left[\tilde{\bar{u}}_{b_x} + \left(3\sqrt{\tilde{\bar{u}}_{b_x}}\right), \tilde{\bar{u}}_{b_y} + \left(3\sqrt{\tilde{\bar{u}}_{b_y}}\right), \tilde{\bar{u}}_{b_z} + \left(3\sqrt{\tilde{\bar{u}}_{b_z}}\right)\right]$$
$$\left(CL_{\tilde{u}_{b_x}}, CL_{\tilde{u}_{b_y}}, CL_{\tilde{u}_{b_z}}\right) = \left(\tilde{\bar{u}}_{b_x}, \tilde{\bar{u}}_{b_y}, \tilde{\bar{u}}_{b_z}\right)$$
$$\left(LCL_{\tilde{u}_{b_x}}, LCL_{\tilde{u}_{b_y}}, LCL_{\tilde{u}_{b_z}}\right) = \left[\tilde{\bar{u}}_{b_x} - \left(3\sqrt{\tilde{\bar{u}}_{b_x}}\right), \tilde{\bar{u}}_{b_y} - \left(3\sqrt{\tilde{\bar{u}}_{b_y}}\right), \tilde{\bar{u}}_{b_z} - \left(3\sqrt{\tilde{\bar{u}}_{b_z}}\right)\right]$$

The proposed and approved standard deviations ($\tilde{\sigma}_{i.\tilde{u}:F-C_p}$, $i = x, y, z$) for $\tilde{u}$-fuzzy control chart are estimated and cautiously evaluated by adopting process capability $C_p = \dfrac{USL_{i.\tilde{u}:F-C_p} - LSL_{i.\tilde{u}:F-C_p}}{6\sigma}, i = x, y, z.$ A JAVA script as per the recommendation is being pertained to evaluate the standard deviations (Radhakrishnan and Balamurugan, 2011) based on $\frac{\sum_{j=1}^{n}\tilde{u}_{b_{ij}}}{n}$, $i = x, y, z$ and $j = 1, 2, ...n.$

So, the results of the proposed fuzzy control limits for average fraction defectives ($\tilde{u}$) with the assist of process capability are as follows:

$$\left(UCL_{\tilde{u}_{b_x}}, UCL_{\tilde{u}_{b_y}}, UCL_{\tilde{u}_{b_z}}\right) = \left[\bar{\tilde{u}}_{b_x} + \left(3\tilde{\sigma}_{x.\tilde{u}:F-C_p}\right), \bar{\tilde{u}}_{b_y} + \left(3\tilde{\sigma}_{y.\tilde{u}:F-C_p}\right), \bar{\tilde{u}}_{b_z} + \left(3\tilde{\sigma}_{z.\tilde{u}:F-C_p}\right)\right]$$

$$\left(CL_{\tilde{u}_{b_x}}, CL_{\tilde{u}_{b_y}}, CL_{\tilde{u}_{b_z}}\right) = \left(\bar{\tilde{u}}_{b_x}, \bar{\tilde{u}}_{b_y}, \bar{\tilde{u}}_{b_z}\right)$$

$$\left(LCL_{\tilde{u}_{b_x}}, LCL_{\tilde{u}_{b_y}}, LCL_{\tilde{u}_{b_z}}\right) = \left[\bar{\tilde{u}}_{b_x} - \left(3\tilde{\sigma}_{x.\tilde{u}:F-C_p}\right), \bar{\tilde{u}}_{b_y} - \left(3\tilde{\sigma}_{y.\tilde{u}:F-C_p}\right), \bar{\tilde{u}}_{b_z} - \left(3\tilde{\sigma}_{z.\tilde{u}:F-C_p}\right)\right]$$

4. Illustration

The Following Table-1 refers to the average number of outlet leaks per radiator for 15 lots of 100 radiators each. The triangular fuzzy numbers are obtained by using computer program based on the below observations and also shown in Table-1.

Table-1: Observation of average number of outlet leaks per radiator

Lot Number	Number of leaks	Leaks per radiator	Triangular fuzzy numbers		
			b_x	b_y	b_z
1	18	0.18	14	18	20
2	20	0.20	16	20	22
3	15	0.15	10	15	18
4	19	0.19	15	19	20
5	17	0.17	12	17	19
6	8	0.08	4	8	9
7	17	0.17	11	17	20
8	14	0.14	9	14	18
9	12	0.12	7	12	14
10	13	0.13	8	13	15
11	10	0.10	5	10	14
12	16	0.16	9	16	19
13	11	0.11	8	11	14
14	7	0.07	4	7	11
15	12	0.12	11	12	20

Then centre line (CL) for the fuzzy $\tilde{u}$ -control chart are as follows:

$$\tilde{\tilde{u}}_{b_{x_j}} = \frac{\sum_{j=1}^{n} \tilde{u}_{b_{x\,j}}}{n} = \frac{1.43}{15} = 0.10$$

$$\tilde{\tilde{u}}_{b_{y_j}} = \frac{\sum_{j=1}^{n} \tilde{u}_{b_{y\,j}}}{n} = \frac{2.09}{15} = 0.15$$

and $$\tilde{\tilde{u}}_{b_{z_j}} = \frac{\sum_{j=1}^{n} \tilde{u}_{b_{z\,j}}}{n} = \frac{2.53}{15} = 0.17$$

The constructed fuzzy centre line, fuzzy upper and fuzzy lower limits of the fuzzy rule $\tilde{u}$ -control chart are given as follows:

$$\left(UCL_{\tilde{u}_{b_x}}, UCL_{\tilde{u}_{b_y}}, UCL_{\tilde{u}_{b_z}}\right) = \begin{bmatrix} \tilde{\tilde{u}}_{b_x} + \left(3\sqrt{\tilde{\tilde{u}}_{b_x}}\right), \\ \tilde{\tilde{u}}_{b_y} + \left(3\sqrt{\tilde{\tilde{u}}_{b_y}}\right), \\ \tilde{\tilde{u}}_{b_z} + \left(3\sqrt{\tilde{\tilde{u}}_{b_z}}\right) \end{bmatrix}$$

$$\left(UCL_{\tilde{u}_{b_x}}, UCL_{\tilde{u}_{b_y}}, UCL_{\tilde{u}_{b_z}}\right) = \begin{bmatrix} 0.10 + \left(3\sqrt{0.10}\right) = 1.022, \\ 0.15 + \left(3\sqrt{0.15}\right) = 1.326, \\ 0.17 + \left(3\sqrt{0.17}\right) = 1.401 \end{bmatrix}$$

$$\left(CL_{\tilde{u}_{b_x}}, CL_{\tilde{u}_{b_y}}, CL_{\tilde{u}_{b_z}}\right) = \left(\tilde{\tilde{u}}_{b_x}, \tilde{\tilde{u}}_{b_y}, \tilde{\tilde{u}}_{b_z}\right)$$

$$\left(CL_{\tilde{u}_{b_x}}, CL_{\tilde{u}_{b_y}}, CL_{\tilde{u}_{b_z}}\right) = \left(0.10, 0.15, 0.17\right)$$

$$\left(LCL_{\tilde{u}_{b_x}}, LCL_{\tilde{u}_{b_y}}, LCL_{\tilde{u}_{b_z}}\right) = \begin{bmatrix} \tilde{\tilde{u}}_{b_x} - \left(3\sqrt{\tilde{\tilde{u}}_{b_x}}\right), \\ \tilde{\tilde{u}}_{b_y} - \left(3\sqrt{\tilde{\tilde{u}}_{b_y}}\right), \\ \tilde{\tilde{u}}_{b_z} - \left(3\sqrt{\tilde{\tilde{u}}_{b_z}}\right) \end{bmatrix}$$

$$\left(LCL_{\tilde{u}_{b_x}}, LCL_{\tilde{u}_{b_y}}, LCL_{\tilde{u}_{b_z}}\right) == \begin{bmatrix} 0.10 - \left(3\sqrt{0.10}\right) = -0.831 \square 0, \\ 0.15 - \left(3\sqrt{0.15}\right) = -1.020 \square 0, \\ 0.17 - \left(3\sqrt{0.17}\right) = -1.063 \square 0 \end{bmatrix}$$

The recommended standard deviations for $\tilde{u}$ -fuzzy control chart are calculated by using process capability and presented in the Table-2.

Table-2: Calculation of proposed standard deviations from triangular fuzzy numbers for average number of fraction defectives ($\tilde{u}$)

Lot number	$\tilde{\sigma}_{x.\tilde{u}:F-C_p}$	$\tilde{\sigma}_{y.\tilde{u}:F-C_p}$	$\tilde{\sigma}_{z.\tilde{u}:F-C_p}$	$\tilde{\sigma}^{\alpha}_{x.\tilde{u}:F-C_p}$	$\tilde{\sigma}^{\alpha}_{z.\tilde{u}:F-C_p}$
1	0.37	0.42	0.45	0.41	0.46
2	0.40	0.45	0.47	0.43	0.48
3	0.32	0.39	0.42	0.36	0.45
4	0.39	0.44	0.45	0.42	0.45
5	0.35	0.41	0.44	0.39	0.45
6	0.20	0.28	0.30	0.26	0.31
7	0.33	0.41	0.45	0.39	0.47
8	0.30	0.37	0.42	0.35	0.45
9	0.26	0.35	0.37	0.32	0.39
10	0.28	0.36	0.39	0.34	0.40
11	0.22	0.32	0.37	0.29	0.41
12	0.30	0.40	0.44	0.37	0.46
13	0.28	0.33	0.37	0.32	0.40
14	0.20	0.26	0.33	0.24	0.37
15	0.33	0.35	0.45	0.34	0.50

The results of the proposed fuzzy control limits for average number of fraction defectives ($\tilde{u}$) with the assist of process capability are as follows:

$$\left(UCL_{\tilde{u}_{b_x}}, UCL_{\tilde{u}_{b_y}}, UCL_{\tilde{u}_{b_z}}\right) = \begin{bmatrix} \tilde{\bar{u}}_{b_x} + \left(3\tilde{\sigma}_{x.\tilde{u}:F-C_p}\right), \\ \tilde{\bar{u}}_{b_y} + \left(3\tilde{\sigma}_{y.\tilde{u}:F-C_p}\right), \\ \tilde{\bar{u}}_{b_z} + \left(3\tilde{\sigma}_{z.\tilde{u}:F-C_p}\right) \end{bmatrix}$$

$$\left(UCL_{\tilde{u}_{b_x}}, UCL_{\tilde{u}_{b_y}}, UCL_{\tilde{u}_{b_z}}\right) = \begin{bmatrix} 0.10 + (3\times 0.0167) = 0.145, \\ 0.15 + (3\times 0.0137) = 0.194, \\ 0.17 + (3\times 0.0141) = 0.211 \end{bmatrix}$$

$$\left(CL_{\tilde{u}_{b_x}}, CL_{\tilde{u}_{b_y}}, CL_{\tilde{u}_{b_z}}\right) = \left(\tilde{\bar{u}}_{b_x}, \tilde{\bar{u}}_{b_y}, \tilde{\bar{u}}_{b_z}\right)$$

$$\left(CL_{\tilde{u}_{b_x}}, CL_{\tilde{u}_{b_y}}, CL_{\tilde{u}_{b_z}}\right) = \left(0.10, 0.15, 0.17\right)$$

$$\left(LCL_{\tilde{u}_{b_x}}, LCL_{\tilde{u}_{b_y}}, LCL_{\tilde{u}_{b_z}}\right) = \begin{bmatrix} \tilde{\bar{u}}_{b_x} - \left(3\tilde{\sigma}_{x.\tilde{u}:F-C_p}\right), \\ \tilde{\bar{u}}_{b_y} - \left(3\tilde{\sigma}_{y.\tilde{u}:F-C_p}\right), \\ \tilde{\bar{u}}_{b_z} - \left(3\tilde{\sigma}_{z.\tilde{u}:F-C_p}\right) \end{bmatrix}$$

$$\left(LCL_{\tilde{u}_{b_x}}, LCL_{\tilde{u}_{b_y}}, LCL_{\tilde{u}_{b_z}}\right) = \begin{bmatrix} 0.10 - (3\times 0.0167) = 0.045, \\ 0.15 - (3\times 0.0137) = 0.112, \\ 0.17 - (3\times 0.0141) = 0.126 \end{bmatrix}$$

5. Conclusion

In statistical process control, some causes such as mental inspection, incomplete data and human judgments in quality characteristic that lead to exist some level of vagueness and uncertainty in attribute control chart, in these situations it is better to apply fuzzy set theory for control charts. Thus, in this paper, we developed a fuzzy average fraction defectives u-chart using process capability to monitor attribute quality characteristic. It is clear that the product/service is not in good quality as expected, accordingly a modification and improvement is needed in the process/system. It is recommended to use proposed fuzzy $\tilde{u}$ -control chart as an alternative to Shewhart control chart.

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