Some results on t-INTUITIONISTIC MULTIFUZZY SUBRING OF A RING

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Abstract: In this paper, we have defined and discussed the notion of t-intuitionistic multi fuzzy subrings(normal subrings) of a ring and its homomorphic behavior with examples.

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Introduction:

After the introduction of fuzzy set by A. Zadeh[21] the notion of intuitionistic fuzzy set (IFS) was introduced by Atanassov [1,2]. The notion of intuitionistic fuzzy subring was introduced by Hur, Kang and Song[4]. After that many researchers tried to gentralize the notion of intuitionistic fuzzy subring. The notion of t- intuitionistic fuzzy subring was intuitionstic by P. K. Sharma[19] and the notion of t-intuitionistic multi fuzzy subset and t-intuitionistic multi fuzzy subgroup were introduced by KR. Balasubramanian and R.Rajangam[3]. Here in this paper, we have studied some properties of t-intuitionistic multi fuzzy subring and t-intuitionistic multi fuzzy normal subring a ring.

PRELIMINARIES:

We first recall some Definition for the sake of completeness of the topic under study.

Definition 2.1[21]

Let X be a non-empty set .A fuzzy subset A of X is defined by a function $A:X\rightarrow[0,1]$.

Definition 2.2:[1,2]

Let X be a fixed non-empty set. An intuitionistic fuzzy set (IFS) A of X is an object of the form $A = \{< x , \mu_A(x) , \nu_A(x) > : x \in X\}$, where $\mu_A : X \rightarrow [0, 1]$ and $\nu_A : X \rightarrow [0, 1]$ define the degree of membership and degree of non-membership of the element $x \in X$ respectively and for any $x \in X$, we have $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 2.3:[13]

A non-empty set R together with two binary operation $*$ and $.$ is called a ring if

(i) R is a group under the binary operation $*$ and

(ii) R is a semigroup under the binary operation $.$.

Definition 2.4:[13]

Let A be an intuitionistic multi fuzzy subset of a ring R. Let t $\in$ [0, 1]. Then the intuitionistic multi fuzzy subset A^t of R is called the t-intuitionistic fuzzy subset of R with respect to intuitionistic multi fuzzy subset A and is defined as $A^t = (\mu_{A^t}, \nu_{A^t})$, where $\mu_{A^t}(r) = min\{\mu_A(r), t\}$ *and* $\nu_{A^t}(r) = max\{\nu_A(r), 1-t\}$ *for all* $r \in R$.

Definition 2.5:[19]

Let $(R,*,.)$ be a ring. Let A be an intuitionistic fuzzy subset of $(R,*,.)$. Let t ∈ [0, 1]. Then the t-intuitionistic fuzzy subset A^t is called t-intuitionistic fuzzy subring (In short t- IFSR) of $(R,*,.)$ if A^t is intuitionistic fuzzy subring of $(R,*,.)$. That is the following conditions hold

(i) $\mu_{A^t}(x * y^{-1}) \geq \min\{\mu_{A^t}(x), \mu_{A^t}(y)\}$ *and* $\nu_{A^t}(x * y^{-1}) \leq \max\{\nu_{A^t}(x), \nu_{A^t}(y)\}$

(ii) $\mu_{A^t}(x.y) \geq \min\{\mu_{A^t}(x), \mu_{A^t}(y)\}$ *and* $\nu_{A^t}(x - y) \leq \max\{\nu_{A^t}(x), \nu_{A^t}(y)$ for all $x, y \in G$

Definition 2.6:[3]

Let A be an intuitionistic multi fuzzy subset of a ring R. Let t $\in$ $[0, 1]^k$. Then the intuitionistic multi fuzzy subset A^t of R is called the t-intuitionistic multi fuzzy subset of ring R with respect to intuitionistic multi fuzzy subset A and is defined as $A^t = (\mu_{A^t}, \nu_{A^t}) = (\{\mu_{A_i^{t_i}}\}, \{\nu_{A_i^{t_i}}\})$,where $\mu_{A_i^{t_i}}$(r) = $min\{\mu_{A_i}(r), t_i\}$ and $\nu_{A_i^{t_i}}$ (r) = $max\{\nu_{A_i}(r), 1-t_i\}$ for all $r \in R$.

Result 2.7:[3]

Let $A^t = (\mu_{A^t}, \nu_{A^t}) = (\{\mu_{A_i^{t_i}}\}, \{\nu_{A_i^{t_i}}\})$ and $B^t = (\mu_{B^t}, \nu_{B^t}) = (\{\mu_{B_i^{t_i}}\}, \{\nu_{B_i^{t_i}}\})$ be two t-intuitionistic multi fuzzy subset of a ring R. Then $(A \cap B)^t = A^t \cap B^t$

Definition 2.8:[3]

Let R_1 and R_2 be any two non-empty sets and $f: R_1 \longrightarrow R_2$ be a mapping. Let A^t and B^t any two t-intuitionistic multi fuzzy sets of R_1 and R_2 respectively having the same dimention k.Then the image of $A^t (\subseteq R_1)$ under the map f is denoted by $f(A^t)$, and is defined as :$\forall y \in R_2$, $f(A^t)(y)$ =$(\mu_{f(A^t)}(y), \nu_{f(A^t)}(y))$ where

$$\mu_{f(A^t)}(y) = \begin{cases} \max\{\mu_{A^t}(x) : x \in f^{-1}(y) \\ 0_k \quad : otherwise \end{cases} \quad \text{and} \quad \nu_{f(A^t)}(y) = \begin{cases} \min\{\nu_{A^t}(x) : x \in f^{-1}(y) \\ 1_k \quad : otherwise \end{cases}$$

That is,

$$f(A^t)(y) = \begin{cases} (\max\{\mu_{A^t}(x) : x \in f^{-1}(y)\} & , \quad \min\{\nu_{A^t}(x) : x \in f^{-1}(y)\})_{i=1}^{k} \\ (0,1)_k & , \quad otherwise \end{cases}$$

where $(0,1)_k = ((0,1),(0,1),\ldots, k\ times$

Also, the $pre-image$ of B^t under the map f is denoted by $f^{-1}(B^t)$ and is defined as : $f^{-1}(B^t)(x) = (\mu_{B^t}(f(x)), \nu_{B^t}(f(x)), \forall x \in R_1$.

Note 2.9:[3] For any x $\in R_1$, we have $\mu_{f(A^t)}(f(x)) \geq \mu_{A^t}(x)$ and $\nu_{f(A^t)}(f(x)) \leq \nu_{A^t}(x)$

Result 2.10:[3]

Let $f : R_1 \rightarrow R_2$ be a mapping and A^t and B^t are two intuitionistic multi fuzzy subset of R_1 and R_2 respectively, then

(i) $f^{-1}(B^t) = (f^{-1}(B))^t$ (ii) $f(A^t) = (f(A))^t\ for\ all\ t \in [0,1]^k$

t-Intuitionistic Multi Fuzzy Subring Of A Ring

Definition 3.1:

Let A be an intuitionistic multi fuzzy subset of order k of a ring $(R,*,.)$. Let t $\in$ [0, 1]^k. Then A^t is called t-intuitionistic multi fuzzy subring (In short t- IMFSR) of $(R,*,.)$ if A^t is intuitionistic multi fuzzy of $(R,*,.)$. That is the following conditions hold

(i)$\mu_{A^t}(x * y^{-1}) \geq \min\{\mu_{A^t}(x), \mu_{A^t}(y)\}$ and $\nu_{A^t}(x * y^{-1}) \leq \max\{\nu_{A^t}(x), \nu_{A^t}(y)\}$

(ii)$\mu_{A^t}(x.y) \geq \min\{\mu_{A^t}(x), \mu_{A^t}(y)\}$ and $\nu_{A^t}(x - y) \leq \max\{\nu_{A^t}(x), \nu_{A^t}(y)$ for all $x, y \in G$. That is,

(i) $\mu_{{A_i}^{t_i}}(x * y^{-1}) \geq min\{\mu_{{A_i}^{t_i}}(x), \mu_{{A_i}^{t_i}}(y)\}$and $\nu_{{A_i}^{t_i}}(x * y^{-1}) \leq max\{\nu_{{A_i}^{t_i}}(x), \nu_{{A_i}^{t_i}}(y)\}$

(ii) $\mu_{{A_i}^{t_i}}(x.y) \geq min\{\mu_{{A_i}^{t_i}}(x), \mu_{{A_i}^{t_i}}(y)\}$ and $\nu_{{A_i}^{t_i}}(x.y) \leq max\{\nu_{{A_i}^{t_i}}(x), \nu_{{A_i}^{t_i}}(y)\}$ for all

$x, y \in G$ and for each i.

Proposition 3.2:

If A is intuitionistic multi fuzzy subring of a ring R, then A is also a t -intuitionistic multi fuzzy subring of $(R,*,.)$.

Proof:

Let x, y $\in R$ be any element of the ring R.

$\mu_{A^t}(x * y^{-1}) = min\ \{\mu_A(x * y^{-1}), t\}$

$\geq \min[min\{\mu_A(x), \mu_A(y^{-1})\}, t]$

$= min\ [min\{\mu_A(x), \mu_A(y)\}, t]$

$=\ min\ [min\{\mu_{A_i}(x), t_i\}, min\{\ \mu_{A_i}(y), t_i\}]$ for each I

$=\ min\ [min\{\mu_A(x), t\}, min\{\mu_A(y), t\}]$

$=\ min\{\ \mu_{A^t}(x), \mu_{A^t}(y)\}$

Thus $\mu_{A^t}(x * y^{-1})\ \geq min\{\mu_{A^t}(x), \mu_{A^t}(y)\}$

And $\nu_{A^t}(x * y^{-1})\ =\ max\ \{\nu_A\ (x * y^{-1}), 1\text{-}t\}$

$\leq\ max\ [max\{\nu_A(x), \nu_A(y^{-1})\}, 1 - t]$

$\leq\ max\ [max\{\nu_A(x), \nu_A(y)\}, 1 - t]$

$=\ max\ [max\{\nu_{A_i}(x), 1\text{-}t_i\}, max\{\ \nu_{A_i}(x), 1\text{-}t_i\}]$, for each i

$=\ max\ [max\{\nu_A(x), 1 -\ t\}, max\{\nu_A(x), 1 -\ t\}]\ =\ max\{\ \nu_{A^t}(x), \nu_{A^t}(y)\}$

Thus $\nu_{A^t}(x * y^{-1})\ \leq\ \ max\{\ \nu_{A^t}(x), \nu_{A^t}(y)\}$. Also we have,

$\mu_{A^t}(x.y) = min\ \{\mu_A(xy), t\}$

$\geq min\ [min\{\mu_A(x), \mu_A(y)\}, t]$

$=\ min\ [min\{\mu_{A_i}(x), t_i\}, min\{\ \mu_{A_i}(y), t_i\}]$ for each I

$=\ min\ [min\{\mu_A(x), t\}, min\{\mu_A(y), t\}]$

$=\ min\{\ \mu_{A^t}(x), \mu_{A^t}(y)\}$

Thus $\mu_{A^t}(x.y)\ \geq min\{\mu_{A^t}(x), \mu_{A^t}(y)\}$

And $\nu_{A^t}(x.y)\ =\ max\ \{\nu_A\ (x.y), 1\text{-}t\}$

$\leq\ max\ [max\{\nu_A(x), \nu_A(y)\}, 1 - t]$

$=\ max\ [max\{\nu_{A_i}(x), 1\text{-}t_i\}, max\{\ \nu_{A_i}(x), 1\text{-}t_i\}]$, for each i

$=\ max\ [max\{\nu_A(x), 1 -\ t\}, max\{\nu_A(x), 1 -\ t\}]\ =\ max\{\ \nu_{A^t}(x), \nu_{A^t}(y)\}$

Thus $\nu_{A^t}(x.y)\ \leq\ \ max\{\ \nu_{A^t}(x), \nu_{A^t}(y)\}$

Hence A is t- intuitionistic multi fuzzy subring of R.

Remark 3.3:

The converse of above **Proposition (3.2)** need not be true

Example 3.4:

Consider the ring $(Z_5, +_5, \times_5)$, where $Z_5 = \{0, 1, 2, 3, 4\}$. Define the Intuitionistic multi fuzzy set A of order two of Z_5 by

$$\mu_A(x) = (\mu_{A_1}(x), \mu_{A_2}(x)) = \begin{cases} (0.7, 0.6) & if\ x = 0 \\ (0.6, 0.5) & if\ x = 2,3 \\ (0.5, 0.4) & if\ x = 1,4 \end{cases} \text{ and}$$

$$\nu_A(x) = (\mu_{A_1}(x), \mu_{A_2}(x)) = \begin{cases} (0.3, 0.4) & if\ x = 0 \\ (0.3, 0.5) & if\ x = 2,3 \\ (0.4, 0.5) & if\ x = 1,4 \end{cases}$$

. Clearly A is not intuitionistic multi fuzzy ring of R. Because $\mu_{A_1}(4) = 0.5 = \mu_{A_1}(2 \times 2) \ngeq \min\{\mu_{A_1}(2), \mu_{A_1}(2)\} = 0.6$. However, if we take $t = (t_1, t_2) = (0.5, 0.4)$ then $\mu_{A^t}(x) = \left(\mu_{{A_1}^{t_1}}(x), \mu_{{A_2}^{t_2}}(x)\right) = (0.5, 0.4)$ and $\nu_{A^t}(x) = \left(\mu_{{A_1}^{t_1}}(x), \mu_{{A_2}^{t_2}}(x)\right) = (0.5, 0.6)$ for all $x \in Z_5$. Now it can be easily proved that A^t is a intuitionistic multi fuzzy ring of Z_5 and hence A^t is a t-intuitionistic multi fuzzy ring of Z_5

Definition 3.5:

Let $(R, *, .)$ be a ring. A t-intuitionistic multi fuzzy subring A^t of R is said to be a t-intuitionistic multi fuzzy normal subring (in short t-IMFNSR) of R if $\mu_{A^t}(x.y) = \mu_{A^t}(x.y)$ and $\nu_{A^t}(x.y) = \nu_{A^t}(y.x)$ for all $x, y \in G$

Proposition 3.6:

Let A be an intuitionistic multi fuzzy subset of ring $(R, *, .)$. let $t = t_i < \ min\{p_i,\ 1 - q_i\}$, where $p_i = \min\{\mu_{A_i}(x): for\ all\ x\ \in\ R\}$ and $q_i = \max\{\nu_{A_i}(x) :\ for\ all\ x\ \in\ R\}$ for all i. Then A is t- intuitionistic multi fuzzy subring of $(R, *, .)$.

Proof :

Since $t_i < \ min\{p_i, 1\text{-}q_i\}$ implies that $p_i > t_i$ and $q_i < 1 - t_i$

$\Rightarrow \min\{\mu_{A_i}(x) :\ \text{for all } x \in R\} > t_i$ and $max\{\nu_{A_i} x) :\ for\ all\ x \in G\} < 1 - t_i$

$\Rightarrow \mu_{A_i}(x) > t_i$ for all $x \in R$ and $\nu_{A_i}(x) < 1 - t_i$ for all $x \in R$ and for each i

$\Rightarrow \mu_A(x) > t\ for\ all\ x \in R$ also $\nu_A(x) < 1 - t\ for\ all\ x \in R$.

That is, $\mu_{A^t}(x * y^{-1}) = \min\{\mu_A(x * y^{-1}), t\}$ and $\nu_{A^t}(x * y^{-1}) = \max\{\nu_A(x * y^{-1}), t\}$. Which gives, $\mu_{A^t}(x * y^{-1}) \geq \min\{\{\mu_A(x), \mu_A(y^{-1})\}, t\}$ and $\nu_{A^t}(x * y^{-1}) \leq \max\{\{\nu_A(x), \nu_A(y^{-1})\}, 1 - t\}$

$\mu_{A^t}(x * y^{-1}) \geq \min\{\min\{\mu_A(x), t\}, \min\{\mu_A(y^{-1})\}, t\}\}$ and

$\nu_{A^t}(x * y^{-1}) \leq \max\{\max\{\nu_A(x), 1 - t\}, \max\{\nu_A(y^{-1}), 1 - t\}\}$. There fore,

$\mu_{A^t}(x * y^{-1}) \geq \min\{\mu_{A^t}(x), \mu_{A^t}(y)\}$ and $v_{A^t}(x * y^{-1}) \leq \max\{v_{A^t}(x), v_{A^t}(y)\}$ hold. Similarly, we have $\mu_{A^t}(x.y) \geq \min\{\mu_{A^t}(x), \mu_{A^t}(y)\}$ and $v_{A^t}(x.y) \leq \max\{v_{A^t}(x), v_{A^t}(y)\}$. Hence A is t-intuitionistic multi fuzzy subring of $(R,*,.)$.

Proposition 3.7:

Intersection of two t- intuitionistic multi fuzzy subring of a ring $(R,*,.)$ is also at- intuitionistic multi fuzzy subring of a ring $(R,*,.)$.

Proof:

Let x, y ∈ R be any element of the ring R. then

$$\mu_{(A\cap B)^t}(x * y^{-1}) = min\{\mu_{(A\cap B)}(x * y^{-1}), t\} = min\{\mu_{(A_i\cap B_i)}(x * y^{-1}), t\}$$

$$= min\{min\{\mu_{A_i}(x * y^{-1}), \mu_{B_i}(x * y^{-1})\}, t_i\}$$

$$= min\,[min\{\mu_{A_i}(x * y^{-1}), t_i\}, min\{\mu_{B_i}(x * y^{-1}), t_i\}]$$

$$= min\{\mu_{A_i{}^{t_i}}(x * y^{-1}), \mu_{B_i{}^{t_i}}(x * y^{-1})\}$$

$$\geq min\,[min\{\mu_{A_i{}^{t_i}}(x), \mu_{A_i{}^{t_i}}(y^{-1})\}, min\{\mu_{B_i{}^{t_i}}(x), \mu_{B_i{}^{t_i}}(y^{-1})\}]$$

$$= min\,[min\{\mu_{A^t}(x), \mu_{A^t}(y)\}, min\{\mu_{B^t}(x), \mu_{B^t}(y)\}]$$

$$= min\,[min\{\mu_{A^t}(x), \mu_{B^t}(x)\}, min\{\mu_{A^t}(y), \mu_{B^t}(y)\}]$$

$$= min\{\mu_{A^t\cap B^t}(x), \mu_{A^t\cap B^t}(y)\}$$

$$= min\{\mu_{(A\cap B)^t}(x), \mu_{(A\cap B)^t}(y)\}$$

Thus $\mu_{(A\cap B)^t}(x * y^{-1}) \geq min\{\mu_{(A\cap B)^t}(x), \mu_{(A\cap B)^t}(y)\}$. Similarly, we have

$$\nu_{(A\cap B)^t}(x * y^{-1}) = max\{\nu_{(A\cap B)}(x * y^{-1}), 1 - t\} = max\{\nu_{(A_i\cap B_i)}(x * y^{-1}), 1 - t_i\}$$

$$= max\{max\{\nu_{A_i}(x * y^{-1}), \nu_{B_i}(x * y^{-1})\}, 1 - t_i\}]$$

$$= max\,[max\{\nu_{A_i}(x * y^{-1}), 1 - t_i\}, max\{\nu_{B_i}(x * y^{-1}), 1 - t_i\}]$$

$$= max\{\nu_{A_i{}^{t_i}}(x * y^{-1}), \nu_{B_i{}^{t_i}}(x * y^{-1})\}$$

$$\leq max\,[max\{\nu_{A_i{}^{t_i}}(x), \nu_{A_i{}^{t_i}}(y^{-1})\}, max\{\nu_{B_i{}^{t_i}}(x), \nu_{B_i{}^{t_i}}(y^{-1})\}]$$

$$= max\,[max\{\nu_{A^t}(x), \nu\mu_{A^t}(y)\}, max\{\nu_{B^t}(x), \nu_{B^t}(y)\}]$$

$$= max\,[max\{\nu(x), \nu_{B^t}(x)\}, max\{\nu_{A^t}(y), \nu_{B^t}(y)\}]$$

$$= max\{\nu_{A^t\cap B^t}(x), \nu_{A^t\cap B^t}(y)\}$$

$$= max\{\nu(x), \nu_{(A\cap B)^t}(y)\}$$

Thus $\nu_{(A\cap B)^t}(x * y^{-1}) \leq max\{\nu_{(A\cap B)^t}(x), \nu_{(A\cap B)^t}(y)\}$.

Similarly we get, $\mu_{(A\cap B)^t}\,(x.y) \geq min\{\mu_{(A\cap B)^t}(x), \mu_{(A\cap B)^t}(y)\}$ and

$\nu_{(A\cap B)^t}\,(x.y) \leq max\{\nu_{(A\cap B)^t}(x), \nu_{(A\cap B)^t}(y)\}$. Hence $A \cap B$ is t- intuitionistic multi fuzzy subring of a ring $(R,*,.)$.

Corollary 3.8:

Intersection of a family of t- intuitionistic multi fuzzy subrings of a ring $(R,*,.)$ is again t- intuitionistic multi fuzzy subring of a ring $(R,*,.)$.

Remark 3.9:

Let A be intuitionistic multi fuzzy normal subrings of a ring $(R,*,.)$, then A is also t- intuitionistic multi fuzzy normal subrings of a ring $(R,*,.)$.

Proof:

Let $x, y \in R$ be any element of the ring R. Then $\mu_{A^t}\,(x.y) = \min\{\mu_A\,(x.y), t\} = \min\{\mu_A\,(y.x), t\} = \mu_{A^t}\,(y.x)$. Similarly, we have, $\nu_{A^t}\,(x.y) = \max\{\nu_A\,(x.y), 1-t\} = \max\{\nu_A\,(y.x), 1-t\} = \nu_{A^t}\,(y.x)$. Hence A is a t- intuitionistic multi fuzzy normal subrings of a ring $(R,*,.)$.

Definition 3.10:

Let A be a intuitionistic multi fuzzy subset of a ring $(R,*,.)$. Let $t \in [0,1]^k$. Then A is called

a) t- intuitionistic multi fuzzy left ideal of R(In short t-IMFLI) if

(i). $\mu_{A^t}\,(x * y^{-1}) \geq min\{\mu_{A^t}(x), \mu_{A^t}(y)\}$ (ii). $\mu_{A^t}\,(x.y) \geq \mu_{A^t}(y)$

(iii). $\nu_{A^t}\,(x * y^{-1}) \leq max\{\nu_{A^t}(x), \nu_{A^t}(y)\}$ (iv). $\nu_{A^t}\,(x.y) \leq \nu_{A^t}(y)$ for all $x, y \in R$

b) t- intuitionistic multi fuzzy right ideal of R(In short t-IMFRI) if

(i). $\mu_{A^t}\,(x * y^{-1}) \geq min\{\mu_{A^t}(x), \mu_{A^t}(y)\}$ (ii). $\mu_{A^t}\,(x.y) \geq \mu_{A^t}(x)$

(iii). $\nu_{A^t}\,(x * y^{-1}) \leq max\{\nu_{A^t}(x), \nu_{A^t}(y)\}$ (iv). $\nu_{A^t}\,(x.y) \leq \nu_{A^t}(x)$ for all $x, y \in R$

c) t- intuitionistic multi fuzzy ideal of R(In short t-IMFI) if

(i). $\mu_{A^t}\,(x * y^{-1}) \geq min\{\mu_{A^t}(x), \mu_{A^t}(y)\}$ (ii). $\mu_{A^t}\,(x.y) \geq \max\{\mu_{A^t}(x), \mu_{A^t}(y)\}$

(iii). $\nu_{A^t}\,(x * y^{-1}) \leq max\{\nu_{A^t}(x), \nu_{A^t}(y)\}$ (iv). $\nu_{A^t}\,(x.y) \leq \min\{\nu_{A^t}(x), \nu_{A^t}(y)\}$ for all $x, y \in R$

Proposition 3.11:

If A is intuitionistic multi fuzzy left ideal of a ring R, then A is also t- intuitionistic multi fuzzy left ideal of R

Proof : By the **Preposition 3.2,** if A is intuitionistic multi fuzzy subring of a ring R, then A is also a t -intuitionistic multi fuzzy subring of R.

Then it is enough to prove that $\mu_{A^t}(xy) \geq \mu_{A^t}(y)$ and $\nu_{A^t}(xy) \leq \nu_{A^t}(y)$, hold for all $x, y \in R$.

$\mu_{A^t}(xy) = \min\{\mu_A(xy), t\} \geq \min\{\mu_A(y), t\} = \mu_{A^t}(y)$ and $\nu_{A^t}(xy) = max\{\nu_A(xy), 1-t\} \leq max\{\nu_A(y), 1-t\} = \nu_{A^t}(y)$, hold for all $x, y \in R$.

Thus, $\mu_{A^t}(xy) \geq \mu_{A^t}(y)$ and $\nu_{A^t}(xy) \leq \nu_{A^t}(y)$. Hence A is t- intuitionistic multi fuzzy left ideal of R.

Proposition 3.12:

If A is intuitionistic multi fuzzy right ideal of a ring R, then A is also t- intuitionistic multi fuzzy right ideal of R

Proof : By the **Preposition 3.2,** if A is intuitionistic multi fuzzy subring of a ring R, then A is also a t -intuitionistic multi fuzzy subring of R. Then it is enough to prove that $\mu_{A^t}(xy) \geq \mu_{A^t}(x)$ and $\nu_{A^t}(xy) \leq \nu_{A^t}(x)$, hold for all $x, y \in R$.

$\mu_{A^t}(xy) = \min\{\mu_A(xy), t\} \geq \min\{\mu_A(x), t\} = \mu_{A^t}(y)$ and $\nu_{A^t}(xy) = \max\{\nu_A(xy), 1-t\} \leq \max\{\nu_A(x), 1-t\} = \nu_{A^t}(x)$, hold for all $x, y \in R$.

Thus, $\mu_{A^t}(xy) \geq \mu_{A^t}(x)$ and $\nu_{A^t}(xy) \leq \nu_{A^t}(x)$. Hence A is t- intuitionistic multi fuzzy right ideal of R.

Proposition 3.13:

If A is intuitionistic multi fuzzy ideal of a ring R, then A is also t- intuitionistic multi fuzzy ideal of R

Proof : By the **Preposition 3.2,** if A is intuitionistic multi fuzzy subring of a ring R, then A is also a t -intuitionistic multi fuzzy subring of R. Then it is enough to prove that $\mu_{A^t}(xy) \geq \max\{\mu_{A^t}(x), \mu_{A^t}(y)\}$ and $\nu_{A^t}(xy) \leq \min\{\nu_{A^t}(x), \nu_{A^t}(y)\}$, hold for all $x, y \in R$.

For any t- intuitionistic multi fuzzy ideal of R satisfies the axioms of both left and right fuzzy ideal of R. Hence by the above two Prepositions, we have $\mu_{A^t}(xy) \geq \mu_{A^t}(y)$, $\nu_{A^t}(xy) \leq \nu_{A^t}(y)$ and $\mu_{A^t}(xy) \geq \mu_{A^t}(x)$, $\nu_{A^t}(xy) \leq \nu_{A^t}(x)$, hold for all $x, y \in R$. Therefore by composing them we get, $\mu_{A^t}(xy) \geq \max\{\mu_{A^t}(x), \mu_{A^t}(y)\}$ and $\nu_{A^t}(xy) \leq \min\{\nu_{A^t}(x), \nu_{A^t}(y)\}$, hold for all $x, y \in R$. Hence A is t- intuitionistic multi fuzzy right ideal of R.

Homomorphism of t –Intuitioistic Multi Fuzzy Groups

Theorem 4.1:

Let $(R_1, +, \times)$ and $(R_2, *, .)$ are two rings and let $f: R_1 \to R_2$ be homomorphism of ring R_1 in to a ring R_2. Let B^t be t- intuitionistic multi fuzzy subring of ring R_2. Then $f^{-1}(B^t)$ is t-intuitionistic multi fuzzy subring of ring R_1.

Proof:

Let B^t be t- intuitionistic multi fuzzy subring of ring R_2. Let $x_1, x_2 \in R_1$ be any two elements. Then $f^{-1}(B^t)(x_1 + x_2{}^{-1}) = \left(\mu_{f^{-1}(B^t)}(x_1 + x_2{}^{-1}), \nu_{f^{-1}(B^t)}(x_1 + x_2{}^{-1})\right)$

$$\mu_{f^{-1}(B^t)}(x_1 + x_2{}^{-1}) = \mu_{f^{-1}\,({B_i}^t)}(x_1 + x_2{}^{-1})$$
$$= \mu_{{B_i}^{t_i}}(f(x_1 + x_2{}^{-1})) = \mu_{{B_i}^{t_i}}(f(x_1) * f(x_2{}^{-1}))$$
$$\geq min\ \{\mu_{{B_i}^{t_i}} f(x_1), \mu_{{B_i}^{t_i}} f(x_2)\} = min\{\mu_{f^{-1}\,({B_i}^{t_i})}(x_1), \mu_{f^{-1}\,({B_i}^{t_i})}(x_2)\}$$
$$= min\{\mu_{f^{-1}(B^t)}(x_1), \mu_{f^{-1}(B^t)}(x_2)\}$$

Thus $\mu_{f^{-1}(B^t)}(x_1 + x_2{}^{-1}) \geq min\,\{\mu_{f^{-1}(B^t)}(x_1), \mu_{f^{-1}(B^t)}(x_2)\}$

Similarly, $\nu_{f^{-1}(B^t)}(x_1 + x_2{}^{-1}) = \nu_{f^{-1}\,({B_i}^{t_i})}(x_1 + x_2{}^{-1})$

$$= \nu_{{B_i}^{t_i}}(f(x_1 + x_2{}^{-1})) = \nu_{{B_i}^{t_i}}(f(x_1) * f(x_2{}^{-1}))$$
$$\leq max\{\nu_{{B_i}^{t_i}} f(x_1) * \nu_{{B_i}^{t_i}} f(x_2)\} = max\{\nu_{f^{-1}\,({B_i}^{t_i})}(x_1), \nu_{f^{-1}\,({B_i}^{t_i})}(x_2)\}$$
$$= max\{\nu_{f^{-1}(B^t)}(x_1), \nu_{f^{-1}(B^t)}(x_2)\}$$

Thus $\nu_{f^{-1}(B^t)}(x_1 + x_2{}^{-1}) \leq max\,\{\nu_{f^{-1}(B^t)}(x_1), \nu_{f^{-1}(B^t)}(x2)\}$. Also we have, for any $x_1, x_2 \in R_1$, Then $f^{-1}(B^t)(x_1 \times x_2{}^{-1}) = \left(\mu_{f^{-1}(B^t)}(x_1 \times x_2{}^{-1}), \nu_{f^{-1}(B^t)}(x_1 \times x_2{}^{-1})\right)$

$$\mu_{f^{-1}(B^t)}(x_1 \times x_2{}^{-1}) = \mu_{f^{-1}\,({B_i}^t)}(x_1 \times x_2{}^{-1})$$
$$= \mu_{{B_i}^{t_i}}(f(x_1 \times x_2{}^{-1})) = \mu_{{B_i}^{t_i}}(f(x_1).f(x_2{}^{-1}))$$
$$\geq min\ \{\mu_{{B_i}^{t_i}} f(x_1), \mu_{{B_i}^{t_i}} f(x_2)\} = min\{\mu_{f^{-1}\,({B_i}^{t_i})}(x_1), \mu_{f^{-1}\,({B_i}^{t_i})}(x_2)\}$$
$$= min\{\mu_{f^{-1}(B^t)}(x_1), \mu_{f^{-1}(B^t)}(x_2)\}$$

Thus $\mu_{f^{-1}(B^t)}(x_1 \times x_2{}^{-1}) \geq min\,\{\mu_{f^{-1}(B^t)}(x_1), \mu_{f^{-1}(B^t)}(x_2)\}$

Similarly, $\nu_{f^{-1}(B^t)}(x_1 \times x_2{}^{-1}) = \nu_{f^{-1}\,({B_i}^{t_i})}(x_1 \times x_2{}^{-1})$

$$= \nu_{{B_i}^{t_i}}(f(x_1 \times x_2{}^{-1})) = \nu_{{B_i}^{t_i}}(f(x_1).f(x_2{}^{-1}))$$
$$\leq max\{\nu_{{B_i}^{t_i}} f(x_1).\nu_{{B_i}^{t_i}} f(x_2)\} = max\{\nu_{f^{-1}\,({B_i}^{t_i})}(x_1), \nu_{f^{-1}\,({B_i}^{t_i})}(x_2)\}$$
$$= max\{\nu_{f^{-1}(B^t)}(x_1), \nu_{f^{-1}(B^t)}(x_2)\}.$$

Thus $\nu_{f^{-1}(B^t)}(x_1 \times {x_2}^{-1}) \leq max\{\nu_{f^{-1}(B^t)}(x_1), \nu_{f^{-1}(B^t)}(x2)\}$. Hence $f^{-1}(B^t)$ is intuitionistic multi fuzzy subring of R_1.

Theorem 4.2:

Let $f: R_1 \to R_2$ be a ring homomorphism from a ring $(R_1, +, \times)$ in to a ring $(R_2, *, .)$. Let B^t be t- intuitionistic multi fuzzy normal subring of ring R_2. Then $f^{-1}(B^t)$ is t- intuitionistic multi fuzzy normal subring of ring R_1.

Proof:

Let B^t be t- intuitionistic multi fuzzy normal subring of ring R_2. Let x_1 and x_2 be any two elements of R_1. We know that $f^{-1}(B^t)(x_1 \times x_2) = \left(\mu_{f^{-1}(B^t)}(x_1 \times x_2), \nu_{f^{-1}(B^t)}(x_1 \times x_2)\right)$. So we need to show that only $\mu_{f^{-1}(B^t)}(x_1 \times x_2) = \mu_{f^{-1}(B^t)}(x_2 \times x_1)$ and $\nu_{f^{-1}(B^t)}(x_1 \times x_2) = \nu_{f^{-1}(B^t)}(x_2 \times x_1)$

$$\mu_{f^{-1}(B^t)}(x_1 \times x_2) = \mu_{B^t}\big(f(x_1 \times x_2)\big) = \mu_{{B_i}^{t_i}}\big(f(x_1 \times x_2)\big)$$

$$= \mu_{{B_i}^{t_i}}\big(f(x_1).f(x_2)\big) = \mu_{{B_i}^{t_i}}\big(f(x_2).f(x_1)\big) = \mu_{B^t}\big(f(x_2).f(x_1)\big)$$

$$= \mu_{B^t}\big(f(x_2 \times x_1)\big) = \mu_{f^{-1}(B^t)}(x_2 \times x_1).$$

And $\nu_{f^{-1}(B^t)}(x_1 \times x_2) = \nu_{B^t}\big(f(x_1 \times x_2)\big) = \nu_{{B_i}^{t_i}}\big(f(x_1 \times x_2)\big)$

$$= \nu_{{B_i}^{t_i}}\big(f(x_1).f(x_2)\big) = \nu_{{B_i}^{t_i}}\big(f(x_2).f(x_1)\big) = \nu_{B^t}\big(f(x_2).f(x_1)\big)$$

$$= \nu_{B^t}\big(f(x_2 \times x_1)\big) = \nu_{f^{-1}(B^t)}(x_2 \times x_1)$$

Hence $f^{-1}(B^t)$ is t- intuitionistic multi fuzzy normal subring of ring R_1.

Theorem 4.3:

Let $f: R_1 \to R_2$ be a ring homomorphism from a ring $(R_1, +, \times)$ in to a ring $(R_2, *, .)$. Let B^t be t- intuitionistic multi fuzzy left ideal of ring R_2. Then $f^{-1}(B^t)$ is t- intuitionistic multi fuzzy left ideal of ring R_1.

Proof:

Let B^t be t- intuitionistic multi fuzzy left ideal of ring R_2. Let x_1 and x_2 be any two elements of R_1. Then in view of Proposition (4.1), we need to show that only $\mu_{f^{-1}(B^t)}(x_1 \times x_2) \geq \mu_{f^{-1}(B^t)}(x_2)$ and $\nu_{f^{-1}(B^t)}(x_1 \times x_2) \leq \nu_{f^{-1}(B^t)}(x_2)$

$$\mu_{f^{-1}(B^t)}(x_1 \times x_2) = \mu_{B^t}\big(f(x_1 \times x_2)\big) = \mu_{{B_i}^{t_i}}\big(f(x_1 \times x_2)\big)$$

$$= \mu_{{B_i}^{t_i}}\big(f(x_1).f(x_2)\big) = \mu_{{B_i}^{t_i}}\big(f(x_2)\big) = \mu_{B^t}\big(f(x_2)\big).$$ That is,

$$\mu_{f^{-1}(B^t)}(x_1 \times x_2) \geq \mu_{f^{-1}(B^t)}(x_2).$$

And $\nu_{f^{-1}(B^t)}(x_1 \times x_2) = \nu_{B^t}\big(f(x_1 \times x_2)\big) = \nu_{B_i{}^{t_i}}\big(f(x_1 \times x_2)\big)$

$= \nu_{B_i{}^{t_i}}\big(f(x_1).f(x_2)\big) = \nu_{B_i{}^{t_i}}\big(f(x_2)\big) = \nu_{B^t}\big(f(x_2)\big)$. That is,

$\nu_{f^{-1}(B^t)}(x_1 \times x_2) \leq \nu_{f^{-1}(B^t)}(x_2)$. Hence $f^{-1}(B^t)$ is t- intuitionistic multi fuzzy left ideal of ring R_1.

Theorem 4.4:

Let $f: R_1 \to R_2$ be a ring homomorphism from a ring $(R_1, +, \times)$ in to a ring $(R_2, *, .)$. Let B^t be t- intuitionistic multi fuzzy right ideal of ring R_2. Then $f^{-1}(B^t)$ is t- intuitionistic multi fuzzy right ideal of ring R_1.

Proof : It can be obtained similar to **Theorem 4.3.**

Theorem 4.5:

Let $f: R_1 \to R_2$ be a ring homomorphism from a ring $(R_1, +, \times)$ in to a ring $(R_2, *, .)$. Let B^t be t- intuitionistic multi fuzzy ideal of ring R_2. Then $f^{-1}(B^t)$ is t- intuitionistic multi fuzzy ideal of ring R_1.

Proof : It follows from **Theorem 4.3** and **Theorem 4.4.**

Theorem 4.6:

Let $(R_1, +, \times)$ and $(R_2, *, .)$ be two rings and $f: R_1 \to R_2$ be a surjective ring homomorphism. Let A^t be t- intuitionistic multi fuzzy subring of ring R_1. Then $f(A^t)$ is t- intuitionistic multi fuzzy subring of ring R_2.

Proof :

Since A^t is t- intuitionistic multi fuzzy subring of ring R_1. Let y_1, y_2 be any two elements of R_2. Then there exist some $x_1, x_2 \in R_1$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$. [Note that x_1, x_2 need not be unique]

$$f(A^t)(y_1 * y_2{}^{-1}) = f(A_i{}^{t_i})(y_1 .* y_2{}^{-1}) = (\mu_{f(A_i{}^{t_i})}(y_1 * y_2{}^{-1}), \nu_{f(A_i{}^{t_i})}(y_1 * y_2{}^{-1}))$$

$$\mu_{f(A^t)}(y_1 .* y_2{}^{-1}) = \mu_{f(A_i{}^{t_i})}(y_1 * y_2{}^{-1}) = \mu_{(f(A_i))^{t_i}}(y_1 * y_2{}^{-1})$$

$$= min\{\mu_{f(A_i)}(f(x_1) * (f(x_2))^{-1}), t_i\}$$

$$= \min\{\mu_{f(A_i)}(f(x_1 + x_2{}^{-1})), t_i\}$$

$$\geq \min\{\mu_{A_i}(x_1 + x_2{}^{-1}), t_i\} = \mu_{A_i{}^{t_i}}(x_1 + x_2{}^{-1})$$

$$\geq \min\{\mu_{A_i{}^{t_i}}(x_1), \mu_{A_i{}^{t_i}}(x_2)\} \text{ for all } x_1, x_2 \in R_1 \text{ such that } f(x_1) = y_1 \text{ and } f(x_2) = y_2$$

$$= \min\{\vee\{\mu_{A_i{}^{t_i}}(x_1) : f(x_1) = y_1,\}, \vee\{\mu_{A_i{}^{t_i}}(x_2) : f(x_2) = y_2\}\}$$

$$= \min\{\mu_{f(A_i{}^{t_i})}(y_1), \mu_{f(A_i{}^{t_i})}(y_2)\} = \min\{\mu_{f(A^t)}(y_1), \mu_{f(A^t)}(y_2)\}$$

Thus $\mu_{f(A^t)}(y_1 * y_2^{-1}) \geq \min\{\mu_{f(A^t)}(y_1), \mu_{f(A^t)}(y_2)\}$.

And $\nu_{f(A^t)}(y_1 * y_2^{-1}) = \nu_{f(A_i{}^{t_i})}(y_1 * y_2^{-1}) = \nu_{(f(A_i))^{t_i}}(y_1 * y_2^{-1})$

$= \max\{\nu_{f(A_i)}(f(x_1) * (f(x_2))^{-1}), t_i\} = \max\{\nu_{f(A_i)}(f(x_1 + x_2^{-1})), t_i\}$

$\leq \max\{\nu_{A_i}(x_1 + x_2^{-1}), t_i\} = \nu_{A_i{}^{t_i}}(x_1 + x_2^{-1})$

$\leq \max\{\nu_{A_i{}^{t_i}}(x_1), \nu_{A_i{}^{t_i}}(x_2)\}$ for all $x_1, x_2 \in R_1$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$

$= \max\{\wedge\{\nu_{A_i{}^{t_i}}(x_1) : f(x_1) = y_1,\}, \wedge\{\nu_{A_i{}^{t_i}}(x_2) : f(x_2) = y_2\}\}$

$= \max\{\nu_{f(A_i{}^{t_i})}(y_1), \nu_{f(A_i{}^{t_i})}(y_2)\} = \max\{\nu_{f(A^t)}(y_1), \nu_{f(A^t)}(y_2)\}$

Thus $\nu_{f(A^t)}(y_1 * y_2^{-1}) \leq \max\{\nu_{f(A^t)}(y_1), \mu_{f(A^t)}(y_2)\}$.

Similarly, $f(A^t)(y_1.y_2) = f(A_i{}^{t_i})(y_1.y_2) = (\mu_{f(A_i{}^{t_i})}(y_1.y_2), \nu_{f(A_i{}^{t_i})}(y_1.y_2))$

$\mu_{f(A^t)}(y_1.y_2) = \mu_{f(A_i{}^{t_i})}(y_1.y_2) = \mu_{(f(A_i))^{t_i}}(y_1.y_2) = \min\{\mu_{f(A_i)}(f(x_1).f(x_2)), t_i\}$

$= \min\{\mu_{f(A_i)}(f(x_1 \times x_2)), t_i\} \geq \min\{\mu_{A_i}(x_1 \times x_2), t_i\} = \mu_{A_i{}^{t_i}}(x_1 \times x_2)$

$\geq \min\{\mu_{A_i{}^{t_i}}(x_1), \mu_{A_i{}^{t_i}}(x_2)\}$ for all $x_1, x_2 \in R_1$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$

$= \min\{\vee\{\mu_{A_i{}^{t_i}}(x_1) : f(x_1) = y_1,\}, \vee\{\mu_{A_i{}^{t_i}}(x_2) : f(x_2) = y_2\}\}$

$= \min\{\mu_{f(A_i{}^{t_i})}(y_1), \mu_{f(A_i{}^{t_i})}(y_2)\} = \min\{\mu_{f(A^t)}(y_1), \mu_{f(A^t)}(y_2)\}$

Thus $\mu_{f(A^t)}(y_1.y_2) \geq \min\{\mu_{f(A^t)}(y_1), \mu_{f(A^t)}(y_2)\}$ and

$\nu_{f(A^t)}(y_1.y_2) = \nu_{f(A_i{}^{t_i})}(y_1.y_2) = \nu_{(f(A_i))^{t_i}}(y_1.y_2) = max\{\nu_{f(A_i)}(f(x_1).f(x_2)), t_i\}$

$= \max\{\nu_{f(A_i)}(f(x_1 \times x_2)), t_i\} \leq \max\{\nu_{A_i}(x_1 \times x_2), t_i\} = \nu_{A_i{}^{t_i}}(x_1 \times x_2)$

$\leq \max\{\nu_{A_i{}^{t_i}}(x_1), \nu_{A_i{}^{t_i}}(x_2)\}$ for all $x_1, x_2 \in R_1$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$

$= \max\{\wedge\{\nu_{A_i{}^{t_i}}(x_1) : f(x_1) = y_1,\}, \wedge\{\nu_{A_i{}^{t_i}}(x_2) : f(x_2) = y_2\}\}$

$= \max\{\nu_{f(A_i{}^{t_i})}(y_1), \nu_{f(A_i{}^{t_i})}(y_2)\} = \max\{\nu_{f(A^t)}(y_1), \nu_{f(A^t)}(y_2)\}$

Thus $\nu_{f(A^t)}(y_1.y_2) \leq \max\{\nu_{f(A^t)}(y_1), \mu_{f(A^t)}(y_2)\}$. Hence $f(A^t)$ is t- intuitionistic multi fuzzy subring of ring R_2.

Theorem 4.7:

Let $(R_1, +, \times)$ and $(R_2, *, .)$ be two rings and $f: R_1 \to R_2$ be a surjective ring homomorphism. Let A^t be t- intuitionistic multi fuzzy normal subring of ring R_1. Then $f(A^t)$ is t- intuitionistic multi fuzzy normal subring of ring R_2.

Proof :

Since A^t is t- intuitionistic multi fuzzy normal subring of ring R_1. Let y_1, y_2 be any two elements of R_2. Then there exist some $x_1, x_2 \in R_1$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$. [Note that x_1, x_2 need not be unique].

Now $f(A^t)(y_1.y_2) = f(A_i{}^{t_i})(y_1.y_2) = (\mu_{f(A_i{}^{t_i})}(y_1.y_2), \nu_{f(A_i{}^{t_i})}(y_1.y_2))$. On the conclusion of Theorem 4.3, we prove only $\mu_{f(A^t)}(y_1.y_2) = \mu_{f(A^t)}(y_2.y_1)$ and $\nu_{f(A^t)}(y_1.y_2) = \nu_{f(A^t)}(y_2.y_1)$

$$\mu_{f(A^t)}(y_1.y_2) = \mu_{f(A_i{}^{t_i})}(y_1.y_2) = \mu_{(f(A_i))^{t_i}}\big((f(x_1).f(x_2)\big)$$

$$= \mu_{(f(A_i))^{t_i}}\big((f(x_1 \times x_2)\big) = \vee\left\{\mu_{A_i{}^{t_i}}(x_1 \times x_2): f(x_1 \times x_2) = y_1.y_2\right\}$$

$$= \mu_{f(A_i{}^{t_i})}\big(f(x_2 \times x_1)\big) = \mu_{f(A_i{}^{t_i})}\big(f(x_2).f(x_1)\big)$$

$$= \mu_{f(A^t)}\big(f(x_2).f(x_1)\big) = \mu_{f(A^t)}(y_2.y_1) \text{ and}$$

$$\nu_{f(A^t)}(y_1.y_2) = \nu_{f(A_i{}^{t_i})}(y_1.y_2) = \nu_{(f(A_i))^{t_i}}\big((f(x_1).f(x_2)\big)$$

$$= \nu_{(f(A_i))^{t_i}}\big((f(x_1 \times x_2)\big) = \wedge\left\{\nu_{A_i{}^{t_i}}(x_1 \times x_2): f(x_1 \times x_2) = y_1.y_2\right\}$$

$$= \nu_{f(A_i{}^{t_i})}\big(f(x_2 \times x_1)\big) = \nu_{f(A_i{}^{t_i})}\big(f(x_2).f(x_1)\big)$$

$$= \nu_{f(A^t)}\big(f(x_2).f(x_1)\big) = \nu_{f(A^t)}(y_2.y_1). \text{ Hence the proof.}$$

Theorem 4.8:

Let $(R_1, +, \times)$ and $(R_2, *, .)$ be two rings and $f: R_1 \rightarrow R_2$ be a bijective ring homomorphism. Let A^t be t- intuitionistic multi fuzzy left ideal of R_1. Then $f(A^t)$ is t- intuitionistic multi fuzzy left ideal of R_2.

Proof :

Since A^t is t- intuitionistic multi fuzzy left ideal of R_1. Let y_1, y_2 be any two elements of R_2. Then there exist unique $x_1, x_2 \in R_1$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Now $f(A^t)(y_1.y_2) = f(A_i{}^{t_i})(y_1.y_2) = (\mu_{f(A_i{}^{t_i})}(y_1.y_2), \nu_{f(A_i{}^{t_i})}(y_1.y_2))$. On the conclusion of Theorem 4.3, we prove only $\mu_{f(A^t)}(y_1.y_2) \geq \mu_{f(A^t)}(y_2)$ and $\nu_{f(A^t)}(y_1.y_2) \leq \nu_{f(A^t)}(y_2)$

$$\mu_{f(A^t)}(y_1.y_2) = \mu_{f(A_i{}^{t_i})}(y_1.y_2) = \mu_{(f(A_i))^{t_i}}\big((f(x_1).f(x_2)\big)$$

$$= \mu_{(f(A_i))^{t_i}}\big((f(x_1 \times x_2)\big) = \mu_{A_i{}^{t_i}}(x_1 \times x_2) \geq \mu_{A_i{}^{t_i}}(x_2) = \mu_{f(A_i{}^{t_i})}f(x_2) = \mu_{f(A^t)}f(x_2) = \mu_{f(A^t)}y_2$$

and $\quad \nu_{f(A^t)}(y_1.y_2) = \nu_{f(A_i{}^{t_i})}(y_1.y_2) = \nu_{(f(A_i))^{t_i}}\big((f(x_1).f(x_2)\big)$

$$= \nu_{(f(A_i))^{t_i}}\big((f(x_1 \times x_2)\big) = \nu_{A_i{}^{t_i}}(x_1 \times x_2) \leq \nu_{A_i{}^{t_i}}(x_2) = \nu_{f(A_i{}^{t_i})}f(x_2) = \nu_{f(A^t)}f(x_2) = \nu_{f(A^t)}y_2$$

Hence $f(A^t)$ is a t- intuitionistic multi fuzzy left ideal of R_2.

Theorem 4.9:

Let $(R_1, +, \times)$ and $(R_2, *, .)$ be two rings and $f: R_1 \to R_2$ be a bijective ring homomorphism. Let A^t be t- intuitionistic multi fuzzy right ideal of R_1. Then $f(A^t)$ is t- intuitionistic multi fuzzy right ideal of R_2.

Proof :

It can be obtained similar to **Theorem 4.8.**

Theorem 4.10:

Let $(R_1, +, \times)$ and $(R_2, *, .)$ be two rings and $f: R_1 \to R_2$ be a bijective ring homomorphism. Let A^t be t- intuitionistic multi fuzzy ideal of R_1. Then $f(A^t)$ is t- intuitionistic multi fuzzy ideal of R_2.

Proof :

It can be obtained from **Theorem 4.8 and Theorem 4.9.**

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