Bulk Viscous Strings in LRS Bianchi Type-I Universe with Constant DP and Variable Λ in General Relativity

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Abstract

Here we have studied LRS Bianchi Type-I string universe with bulk viscous fluid and negative constant deceleration parameter and Variable cosmological constant Λ term in general relativity. To find the solutions of survival field equations here we assumed that the shear scalar is proportional to the scalar expansion $\sigma \propto \theta$, deceleration parameter (DP) to be a negative constant quantity and $\Lambda \propto H$. The geometrical as well as physical properties of the model universe are discussed. The present model universe was started with big bang at initial epoch, t=0 with zero volume and then expand with acceleration. The model universe obtained here is shear free. The coefficient of bulk viscosity plays a significant role in the cosmological consequences. In the evolution of universe the tension density vanishes in the late time leaving only particle showing that the present universe is dominated by the particles.

Keywords:- Cloud String, LRS Bianchi Type-I Metric, Bulk Viscosity, General Relativity

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I. INTRODUCTION

In recent year the investigation on cosmic string have becoming considerable interested area for the researchers due to diverse components such that the examine of the universe at early stage, its evolution and expansion of the universe. Grand unified theories(Everett[1], Vilenkin[2]) predicted that strings arose after the explosion of big bang at the time of phase transition when the temperature reached to a value less than some critical temperature. The two authors who started to study relativistically on the string were Letelier[3] and Stachel[4]. Letelier solved the field equations given by Einstein for a cloud of strings with different symmetry such as cylindrical, spherical, and plane.

The Einstein's equation in general relativity without Λ term only admits non static solutions and could not explained the unsolved problem such as the exact age of universe, formations of structures and reason behind the expansion of universe etc. satisfactorily. So to obtain a static solution as accordingly the cosmological principle, Einstein included the Λ term into his field equation in the year 1917. The term Λ can also act as a vacuum energy density. In cosmology, so far many authors have constructed the models with Λ term in different Bianchi Type universe. Singh et al.[5] Constructed a model with Λ and G in Bianchi type-III Universe. A Bianchi type-I universe with massive string , bulk viscosity and vacuum energy density was investigated by Bali and Bola[6]. Pradhan[7] studied a magnetized string cosmological model in anisotropic Bianchi type-I universe with Λ. Also Soni and Shrimali[8,9], Sharma and Sharma[10], Dubey et al.[11] are some among the prominent authors who have discussed various Bianchi type universe in general relativity with Λ term.

The bulk viscosity played a tremendous role at the time of evolution of universe at early stage. It was formed due to different circumstances in the early universe and it results a mechanism for the formation of galaxy. The amount of the bulk viscous stress with respect to expansion can be determined by means of the coefficients of bulk viscosity. At the initial stage of cosmological evolution, some specific aspects of the universe may seem to be appear because of the dissipative process which is caused by viscous fluid. Many authors have discussed different models in Bianchi type cosmology by considering viscous effect. Misner[12,13], studied about the consequences of bulk viscosity in the cosmological evolution of universe. The important of viscous fluid in the cosmological evolution of universe at the early epoch was investigated by Nightingale[14]. Banerjee et al.[15] constructed a Bianchi type-I universe with the bulk viscous fluid. Wang X.X.[16], Bali and Pradhan[17], Kandalkar et al.[18,19], Varun Humad et al.[20], Rao et al.[21], Singh[22], Tripathi et al.[23] are some among the prominent authors who discussed various Bianchi type universe in different space-time with bulk viscosity.

Above literature inspired us to investigate an LRS Bianchi type-I universe with negative constant Deceleration Parameter and variable Λ term in bulk viscous string in general relativity. In 2nd Sec. of this article, an LRS Bianchi type-I metric is presented and the field equations are derived for bulk viscosity and Λ term. Survival field equations are solved in 3rd Sec. In 4th Sec. we discussed the properties of model universe and then concluding remarks are given in last section.

II. Derivation of Field Equations

LRS Bianchi type-I metric can be considered as

$$ds^2 = a^2 dx^2 + b^2 (dy^2 + dz^2) - dt^2 \tag{1}$$

Here a , b are metric functions of 't' alone.

The Einstein's field equations with Λ term and $\frac{8\pi G}{c^4} = 1$ is

$$R_i^j - \frac{1}{2} g_i^j R + \Lambda g_i^j = -T_i^j \tag{2}$$

The energy-momentum tensor with string and bulk viscosity is taken as

$$T_i^j = \rho u_i u^j - \lambda x_i x^j - \xi\theta(u_i u^j + g_i^j) \tag{3}$$

Where ρ and λ are the energy density, tension density respectively and they satisfy the equation $\rho = \rho_p + \lambda$, where ρ_p is particle density. The co-ordinates are co-moving, x_i is a unit vector which is space-like along the strings' direction and u_i represent the four velocity vectors which satisfies the conditions given bellow.

$$u_i u^i = -1 = -x_i x^i \quad \text{and} \quad u_i x^i = 0 \tag{4}$$

$$x^i = (a^{-1},0,0,0) \text{ and } u^i = (0,0,0,1) \tag{5}$$

If R(t) denote the Scale factor, then the volume V is

$$V = ab^2 = R^3 \tag{6}$$

The expansion scalar is

$$\theta = \frac{\dot{a}}{a} + 2\frac{\dot{b}}{b} \tag{7}$$

Hubble parameter H is

$$H = \frac{1}{3}\left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b}\right) \tag{8}$$

The shear scalar is given by

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left[\left(\frac{\dot{a}}{a}\right)^2 + 2\left(\frac{\dot{b}}{b}\right)^2\right] - \frac{\theta^2}{6} \tag{9}$$

And mean anisotropy parameter is

$$\Delta = \frac{1}{3}\sum_{k=1}^{3}\left(\frac{H_k - H}{H}\right)^2 \tag{10}$$

Where, H_k (k=1,2,3) are defined as $H_1 = \frac{\dot{a}}{a}$, and $H_2 = H_3 = \frac{\dot{b}}{b}$ for the metric (1). And they represent directional Hubble Parameters along the directions of axes.

Using the equations (3)-(5) in the equation (2), for the equation (1) yields

$$\frac{\ddot{a}}{a} + \frac{\dot{a}\dot{b}}{ab} + \frac{\ddot{b}}{b} = \xi\theta - \Lambda \tag{11}$$

$$2\frac{\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} = \lambda + \xi\theta - \Lambda \tag{12}$$

$$\frac{\dot{b}^2}{b^2} + \frac{\dot{a}\dot{b}}{ab} = \rho - \Lambda \tag{13}$$

Here, the dots denote the order of differentiation w. r. t.. time.

III. **Solution of the Field Equations**

There are 3 highly nonlinear independent equations from (11)-(13) with five unknowns variables λ, ρ, ξ, Λ, a and b. So to find the exact solutions of the above equations we must used three extra plausible conditions. So here we used the following three physically plausible conditions:

Berman's[24] suggestion regarding variation of Hubble's parameter H provides us a model universe that expands with constant deceleration parameter. So for the determinate solution, let us take deceleration parameter to be a negative constant-

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = h\,(\text{constant}) \qquad (14)$$

It is well known that when q is negative then the model universe expand with acceleration, and when q positive then it explains a decelerating(contracting) universe. Although the present observations like CMBR and SNe Ia suggested the negative value of q but it can be remarkably state that they are not able to deny about the decelerating expansion(positive q) of universe.

The shear scalar and the scalar expansion are proportional, $\sigma \propto \theta$, leading to the equation

$$b = a^n \qquad (15)$$

Here n is a non zero constant.

The above assumption is based on investigation of velocity and red-shift relative for an extragalactic source which predicted that the expansion of Hubble Parameter is 30 percent isotropic, which is supported by the works of Thorne[25]. In particular, it can be said that $\frac{\sigma}{H} \geq 0.30$, where σ and H are respectively shear scalar and Hubble constant respectively. Also, Collins et al.[26] has shown that if the normal to the spatially homogeneous line element is congruent to the homogeneous hyper-surface then $\frac{\sigma}{\theta}$ = constant, θ being the expansion factor.

For the complete determinate solution we also assumed that the cosmological constant Λ is directly proportional to H.

$$\Lambda = lH \qquad (16)$$

Here, l is proportionality constant.

Solving (14), we get

$$R = (r_1 t + r_2)^{\frac{1}{1+h}} \qquad (17)$$

Where r_1 and r_2 are constants of integration.

Using (6), (15) and (17) we get,

$$a = (r_1 t + r_2)^{\frac{3}{(1+h)(2n+1)}} \qquad (18)$$

With the suitable choice of coordinates and constant we can take $r_1 = 1$ and $r_2 = 0$ and then

$$a = t^{\frac{3}{(1+h)(2n+1)}} \quad \text{and} \quad b = t^{\frac{3n}{(1+h)(2n+1)}} \qquad (19)$$

The geometry of the model is given by

$$ds^2 = -dt^2 + t^{\frac{6}{(2n+1)(1+h)}} dx^2 + t^{\frac{6n}{(2n+1)(1+h)}} (dy^2 + dz^2) \qquad (20)$$

IV. Some Physical and Geometric Features

There are 3 highly nonlinear independent equations from (11)-(13) with five unknowns variables λ, ρ, ξ, Λ, a and b. So to find the exact solutions of the above equations we must used three extra plausible conditions. So here we used the following three physically plausible conditions:

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V. Physical Interpretations of the solutions

The model given by the equation (20) represents bulk viscous LRS Bianchi type-I universe with strings in general relativity with constant DP and Λ term. Properties of the model universe for $-1 < h < 0$ are discussed in the following section with help of figures drawn by taking $l = 1$, $n = 2$, $h = -\frac{1}{2}$.

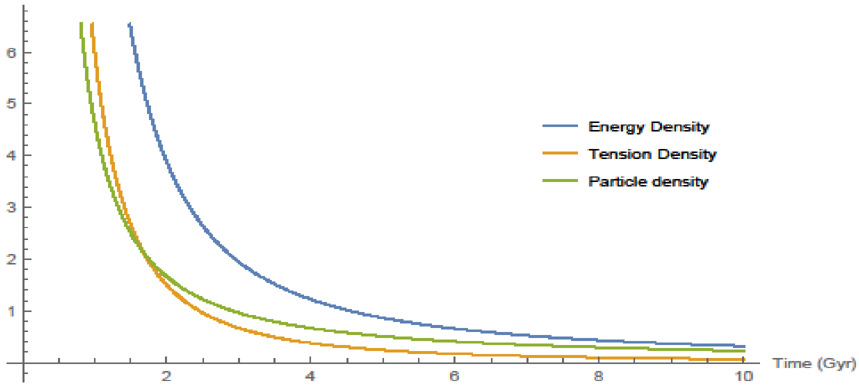

Figure1.Variation of ρ, λ and ρ_p Vs. Time't'.

From fig1. it is seen that energy density $\rho \to \infty$, tension density $\lambda \to \infty$ and particle density $\rho_p \to \infty$ as $t = 0$, and all of them vanish as $t \to \infty$, which indicates that the universe starts at the time $t = 0$. Hence at $t = 0$, this model admits initial singularity and the model satisfies the conditions of energy density i.e., $\rho \geq 0$ and $\rho_p \geq 0$. From the comparison of ρ_p and λ it is observed that, in the late time $\frac{\rho_p}{|\lambda|} > 1$, which shows that in the late time the string diminishes leaving particle density showing the late universe as particle dominated.

Initially at $t = 0$, the volume tends to 0 and it increases gradually with respect to time and $V \to \infty$ as $t \to \infty$, which is shown in fig2. This tells us that the universe is continuously expanding with the passes of time.

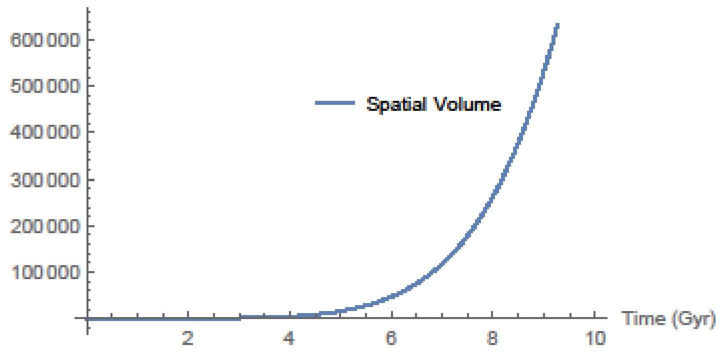

Figure2.Variation of V Vs. Time 't'..

The bulk viscosity $\xi \to \infty$ when $t = 0$ and it decreases with the increases of time and finally when $t \to \infty$ bulk viscosity ξ vanishes(fig3).

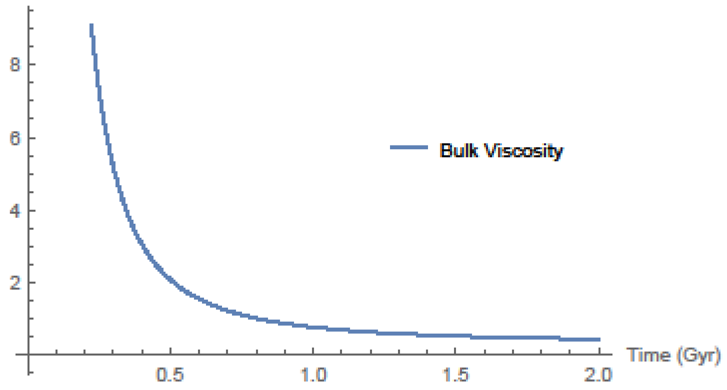

Figure3.Variation of ξ Vs. Time 't'.

At $t = 0$ the θ and H both tend to infinite and as the time progresses gradually they decreases and finally they become 0 when t is infinite(fig4). And since $\frac{dH}{dt}$ is negative quantity so the universe is expanding with acceleration but the expansion rate becomes slower with time increases of time and it stops at $t \to \infty$.

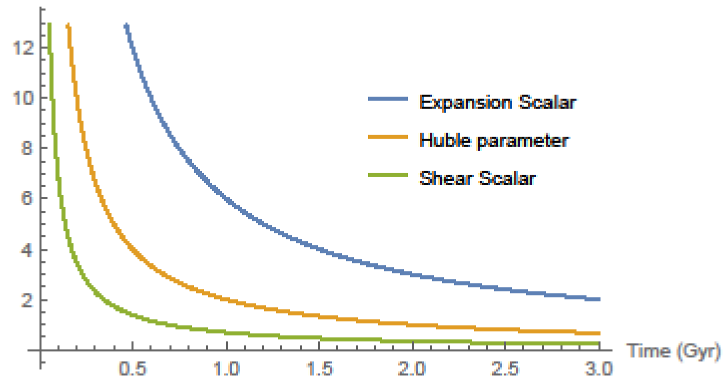

Figure4.Variation of θ, H and σ Vs. Time 't'.

From fig4 it can be seen that $\sigma \to \infty$ at initial epoch and decreases with time and $\sigma \to 0$ at late universe showing that the universe obtained here is non- shearing in the late time.

Here $\frac{\sigma^2}{\theta^2} \neq 0$ (Constant) as $t \to \infty$ for $n \neq 1$ and $\frac{\sigma^2}{\theta^2} = 0$ for $n = 1$. Also From the mean anisotropy parameter $\Delta \neq 0$ (constant) for $n \neq 1$ and $\Delta = 0$ for $n = 1$. From which we can come conclude that for large vale of t the model universe is anisotropic when $n \neq 1$ and it is isotropic for $n = 1$ throughout evolution.

VI. Conclusions

Here we have investigated an LRS Bianchi Type-I bulk viscous string universe with negative constant DP and Variable Λ term in general relativity. The survival field equations are solved by assuming that the shear scalar and expansion scalar is directly proportional to each other $\sigma \propto \theta$, DP to be a negative constant quantity and $\Lambda \propto H$. The geometrical as well as physical aspects of the model universe are discussed. The present universe was started with big bang at initial epoch, t=0 with 0 volume and then expand with acceleration. The model universe obtained here is shear free. The coefficient of bulk viscosity plays an essential role in the cosmological consequences. In the evolution of universe the tension density vanishes in the late time leaving only particle showing that the present universe is dominated by the particles.

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