Multi Intuitionistic Fuzzy Soft Matrix Theory

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Abstract

In this paper, a new type of matrix namely Multi Intuitionistic Fuzzy soft Matrix was introduced and some of its properties are studied. Also we have defined some basic operators on this matrix. We define transpose and complement on this matrix and studied some of their properties. Also we have defined the trace of Multi Intuitionistic Fuzzy Soft Matrix and some of its properties are studied. The concepts are illustrated with suitable numerical examples.

Key Words: Soft Set, Fuzzy Soft Set, Intuitionistic Fuzzy soft set, Multi-Fuzzy Soft Set.

1. INTRODUCTION

In real world problems we have uncertainties. Zadeh [11] in 1965, has introduced the concept namely Fuzzy sets to deal uncertainties which consists of degree of membership. Intuitionistic Fuzzy Sets are introduced by Atanasov[1,2] which are extension of Fuzzy Sets and consists of both membership value and non-membership value associated with every element. The concept Soft set theory have been introduced by Molodtsor [8] in 1999 and he also studied various properties of soft set. Representation of Soft sets in matrix form was given by Cagman et.al [5]. Maji et. Al. [7] have introduced the concept of Intuitionistic fuzzy soft set. Multi sets and Multi Fuzzy Sets were studied in [3,4] and [10]. Intuitionistic Multi fuzzy soft sets were introduced by Sujit Das and Samarjit Kar [9].

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2. PRELEMINARIES

In this section we have given some basic definitions and properties which are required for this paper.

Definition 2.1

Let X denotes a Universal set. Then the membership function μ_A by which a fuzzy set(FS) A is usually defined has the form $\mu_A : X \to [\,0, 1\,]$, where [0 , 1] denotes the interval of real numbers from 0 to 1 inclusive.

Definition 2.2

An Intuitionistic Fuzzy Set (IFS) A in E is defined as an object of the following form $A= \{(x, \mu_A (x), \nu_A (x))/ x \in E\}$ where the functions, $\mu_A : E \to [\,0, 1\,]$ and $\nu_A : E \to [\,0, 1\,]$ define the degree of membership and the degree of non-membership of the element $x \in E$ respectively and for every $x \in E$: $0 \le \mu_A (x) + \nu_A (x) \le 1$.

Definition 2.3

Let U be an initial Universe Set and E be the set of parameters. Let A⊆E. A pair (F,A) is called Fuzzy Soft Set over U where F is a mapping given by $F:A\rightarrow I^U$, where I^U denotes the collection of all fuzzy subsets of U. An fuzzy soft set is a parameterized family of fuzzy subsets of Universe U.

Definition 2.4

Let IP(U) denotes the set of all intuitionistic fuzzy set of U. A pair (F,A) is called a intuitionistic fuzzy soft set (IFSS) of over U, where F is a mapping given by $F:A\rightarrow IP(U)$. For any parameter $e \in A$, F(e) is an intuitionistic fuzzy subset of U and is called Intuitionistic fuzzy value set of parameter e. Clearly F(e) can be written as an Intuitionistic fuzzy set such that $F(e) = \{ x, \mu_{F(e)}(x), \nu_{F(e)}(x) \mid x \in U \}$. Here $\mu_{F(e)}(x)$, $\nu_{F(e)}(x)$ are membership amd non-membership functions respectively and $\forall x \in U$, $\mu_{F(e)}(x) + \nu_{F(e)}(x) \leq 1$,

Definition 2.5

Let IMFS(U) denotes the set of all intuitionistic multi fuzzy set of U. A pair (F,A) is called a intuitionistic multi-fuzzy soft set (IMFSS) of dimension k over U, where F is a mapping given by $F:A\rightarrow IMFS^K(U)$. An intuitionistic multi fuzzy soft set is a mapping from parameters A to $IMFS^K(U)$. It is a parameterized family of intuitionistic multi fuzzy subsets of U. For $e \in A$, F(e) may be considered as the set of e- approximate elements of the intuitionistic multi fuzzy soft set(F,A).

3.Multi Intuitionistic Fuzzy Soft Matrix

In this section, a new matrix namely Multi Intuitionistic Fuzzy Soft Matrix is introduced and various types of this matrices are discussed. Also some of the properties of Multi Intuitionistic Fuzzy Soft Matrices are studied.

Definition 3.1

Let $U = \{u_1,u_2,..., u_m\}$ be the universal set and $E = \{e_1,e_2,..., e_n\}$ be the set of parameters. Let A⊆E and be a Multi Intuitionistic Fuzzy Soft Set on U. Then the matrix associated with this set namely Multi Intuitionistic Fuzzy Soft Matrix $\widetilde{A}^{(K)} = [a_{ij}^{(K)}]_{m\times n}$, i=1,2,…,m j=1,2,…,n

$$\text{where, } a_{ij}^{(K)} = \begin{cases} \left(\mu_j^{(K)}(u_i), \nu_j^{(K)}(u_i)\right) \text{ (or) } \left(\mu_{\tilde{A}_{ij}}^{(K)}, \nu_{\tilde{A}_{ij}}^{(K)}\right) & \text{if } e_j \in A \\ \left(0^{(K)}, 1^{(K)}\right) & \text{if } e_j \notin A \end{cases}$$

Also $0 \leq \mu_j^{(K)}(u_i) + \nu_j^{(K)}(u_i) \leq 1$ and $\mu_j^{(K)}(u_i)$, $\nu_j^{(K)}(u_i)$ and $\mu_{\tilde{A}_{ij}}^{(K)}$, $\nu_{\tilde{A}_{ij}}^{(K)}$ represents the membership and non-membership of the Multi Intuitionistic Fuzzy Soft Set.

The set of all $m \times n$ Multi Intuitionistic Fuzzy Soft Matrices are denoted by $M^{(K)}$IVFSM.

Example 3.2

Suppose that U = $\{u_1,u_2,u_3\}$ is the universal set of students and E = $\{e_1,e_2\}$ is the set of parameters where e_1 = Academic performance and e_2 = Sports performance. Let A = E. Then the Multi Intuitionistic Fuzzy Soft Set (F,A), where F : n → $M^{(K)}IFS$ (Multi Intuitionistic Fuzzy Sets on U) and is given by

F(A) = { F(e_1) = {(u_1, (0.8,0.1),(0.7,0.2))}, {(u_2, (0.5,0.3),(0.4,0.2))}, {(u_3, (0.6,0.2),(0.8,0.1))}} , F(e_2) = {(u_1, (0.7,0.2),(0.6,0.3))}, {(u_2, (0.4,0.2),(0.5,0.1))}, {(u_3, (0.7,0.1),(0.6,0.2))}}}

We can represent the above Multi Intuitionistic Fuzzy Soft Set in Matrix as follows.

$$\widetilde{A}^{(K)}_{3\times 2} = \begin{matrix} & e_1 & e_2 \\ u_1 \\ u_2 \\ u_3 \end{matrix} \begin{pmatrix} ((0.8,0.1),(0.7,0.2)) & ((0.7,0.2),(0.6,0.3)) \\ ((0.5,0.3),(0.4,0.2)) & ((0.4,0.2),(0.5,0.1)) \\ ((0.6,0.2),(0.8,0.1)) & ((0.7,0.1),(0.6,0.2)) \end{pmatrix}_{3\times 2}$$

Definition 3.3

Let $\widetilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ and $\widetilde{B}^{(K)} = [b_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$. Then $\widetilde{A}^{(K)}$ is a Multi Intuitionistic Fuzzy Soft Sub Matrix of $\widetilde{B}^{(K)}$, denoted by $\widetilde{A}^{(K)} \subseteq \widetilde{B}^{(K)}$ if $\mu^{(K)}_{\widetilde{A}_{ij}} \leq \mu^{(K)}_{\widetilde{B}_{ij}}$ and $\nu^{(K)}_{\widetilde{A}_{ij}} \geq \nu^{(K)}_{\widetilde{B}_{ij}}$ for all i,j and K.

Definition 3.4

A Multi Intuitionistic Fuzzy Soft Matrix of order $m \times n$ with cardinality K is called Multi Intuitionistic Fuzzy Soft Null (Zero) Matrix if all of its elements are $\left(0^{(K)},1^{(K)}\right)$. It is denoted by $\tilde{\Phi}^{(K)}$.

Definition 3.5

A Multi Intuitionistic Fuzzy Soft Matrix of order $m \times n$ with cardinality K is called Multi Intuitionistic Fuzzy Soft Absolute Matrix if all of its elements are $\left(1^{(K)},0^{(K)}\right)$. It is denoted by $\tilde{I}^{(K)}$.

Definition 3.6

Let $\widetilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ where $a_{ij}{}^{(K)} = (\mu^{(K)}_{\tilde{A}_{ij}}, \nu^{(K)}_{\tilde{A}_{ij}})$. Then $\widetilde{A}^{(K)}$ is called a Multi Intuitionistic Fuzzy Soft Rectangular Matrix if $m \neq n$.

Definition 3.7

Let $\widetilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ where $a_{ij}{}^{(K)} = (\mu^{(K)}_{\tilde{A}_{ij}}, \nu^{(K)}_{\tilde{A}_{ij}})$. Then $\widetilde{A}^{(K)}$ is called a Multi Intuitionistic Fuzzy Soft Square Matrix if $m = n$.

Definition 3.8

Let $\widetilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ where $a_{ij}{}^{(K)} = (\mu^{(K)}_{\tilde{A}_{ij}}, \nu^{(K)}_{\tilde{A}_{ij}})$. Then $\widetilde{A}^{(K)}$ is called a Multi Intuitionistic Fuzzy Soft Diagonal Matrix if $m = n$.

$$a_{ij}{}^{(K)} = \begin{cases} (\mu^{(K)}_{\tilde{A}_{ij}}, \nu^{(K)}_{\tilde{A}_{ij}}) & \text{if } i = j \\ \left(0^{(K)}, 1^{(K)}\right) & \text{if } i \neq j \end{cases}$$

Definition 3.9

Let $\widetilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ be a Multi Intuitionistic Fuzzy Soft Diagonal Matrix.Then $\widetilde{A}^{(K)}$ is called a Multi Intuitionistic Fuzzy Soft Scalar Matrix if all of its diagonal elements are equal.

Definition 3.10

Let $\widetilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ where $a_{ij}{}^{(K)} = (\mu_j^{(K)}(u_i), \nu_j^{(K)}(u_i))$. Then $\widetilde{A}^{(K)}$ is called a Multi Intuitionistic Fuzzy Soft Upper Triangular Matrix if $m = n$ and

$$a_{ij}{}^{(K)} = \begin{cases} (\mu_j^{(K)}(u_i), \nu_j^{(K)}(u_i)) & \text{if } i \leq j \\ \left(0^{(K)}, 1^{(K)}\right) & \text{if } i > j \end{cases}$$

Definition 3.11

Let $\widetilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ where $a_{ij}{}^{(K)} = (\mu_j^{(K)}(u_i), \nu_j^{(K)}(u_i))$. Then $\widetilde{A}^{(K)}$ is called a Multi Intuitionistic Fuzzy Soft Lower Triangular Matrix if $m = n$ and

$$a_{ij}{}^{(K)} = \begin{cases} (\mu_j^{(K)}(u_i), \nu_j^{(K)}(u_i)) & \text{if } i \geq j \\ \left(0^{(K)}, 1^{(K)}\right) & \text{if } i < j \end{cases}$$

4.Operations on Multi Intuitionistic Fuzzy Soft Matrices

In this section , various operations on Multi Intuitionistic Fuzzy Soft Matrices are defined. Also some of the properties of Multi Intuitionistic Fuzzy Soft Matrices based on these operations are studied and are illustrated with example.

Definition 4.1

If $\widetilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ and $\widetilde{B}^{(K)} = [b_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ then Addition of two Multi Intuitionistic Fuzzy Soft Matrices $\widetilde{A}^{(K)}$ and $\widetilde{B}^{(K)}$ id defined as

$$\widetilde{A}^{(K)} + \widetilde{B}^{(K)} = \left[\left(\max(\mu^{(K)}_{\widetilde{A}_{ij}}, \mu^{(K)}_{\widetilde{B}_{ij}}), \min(\nu^{(K)}_{\widetilde{A}_{ij}}, \nu^{(K)}_{\widetilde{B}_{ij}})\right)\right] \text{ for all i,j and K.}$$

Definition 4.2

If $\widetilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ and $\widetilde{B}^{(K)} = [b_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ then Subtracting of two Multi Intuitionistic Fuzzy Soft Matrices $\widetilde{A}^{(K)}$ and $\widetilde{B}^{(K)}$ id defined as

$$\widetilde{A}^{(K)} - \widetilde{B}^{(K)} = \left[\left(\min(\mu^{(K)}_{\widetilde{A}_{ij}}, \mu^{(K)}_{\widetilde{B}_{ij}}), \max(\nu^{(K)}_{\widetilde{A}_{ij}}, \nu^{(K)}_{\widetilde{B}_{ij}})\right)\right] \text{ for all i,j and K.}$$

Definition 4.3

If $\widetilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ and $\widetilde{B}^{(K)} = [b_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ then Multiplication of two Multi Intuitionistic Fuzzy Soft Matrices $\widetilde{A}^{(K)}$ and $\widetilde{B}^{(K)}$ id defined as

$$\widetilde{A}^{(K)} * \widetilde{B}^{(K)} = [C_{il}]_{m\times p} = \left[\left(\max_{l}\min_{ij}(\mu^{(K)}_{\widetilde{A}_{ij}}, \mu^{(K)}_{\widetilde{B}_{jl}}), \min_{l}\max_{ij}(\nu^{(K)}_{\widetilde{A}_{ij}}, \nu^{(K)}_{\widetilde{B}_{jk}})\right)\right].$$

Example 4.4

Consider $\widetilde{A}_{2\times 2}{}^{(2)} = \begin{pmatrix} ((0.7,0.2)(0.8,0.1)) & ((0.6,0.2)(0.7,0.1)) \\ ((0.5,0.4)(0.4,0.5)) & ((0.2,0.6)(0.4,0.5)) \end{pmatrix}$ and

$$\widetilde{B}_{2\times 2}{}^{(2)} = \begin{pmatrix} ((0.6,0.2)(0.5,0.4)) & ((0.6,0.1)(0.7,0.2)) \\ ((0.7,0.1)(0.6,0.2)) & ((0.5,0.3)(0.2,0.7)) \end{pmatrix}$$

are two Multi Intuitionistic Fuzzy Soft Matrices then

$$\widetilde{A}^{(K)} + \widetilde{B}^{(K)} = \begin{pmatrix} ((0.6,0.2)(0.5,0.4)) & ((0.6,0.1)(0.7,0.1)) \\ ((0.7,0.1)(0.6,0.2)) & ((0.5,0.3)(0.4,0.5)) \end{pmatrix}$$

$$\widetilde{A}^{(K)} - \widetilde{B}^{(K)} = \begin{pmatrix} ((0.6,0.2)(0.5,0.4)) & ((0.6,0.2)(0.7,0.2)) \\ ((0.5,0.4)(0.4,0.5)) & ((0.2,0.6)(0.2,0.7)) \end{pmatrix}$$

$$\widetilde{A}^{(K)} * \widetilde{B}^{(K)} = \begin{pmatrix} ((0.6,0.2)(0.6,0.2)) & ((0.6,0.2)(0.7,0.2)) \\ ((0.5,0.4)(0.4,0.5)) & ((0.5,0.4)(0.4,0.5)) \end{pmatrix}$$

Remark 4.5

In general $\widetilde{A}^{(K)} * \widetilde{B}^{(K)} \neq \widetilde{B}^{(K)} * \widetilde{A}^{(K)}$.

Proposition 4.6

Let $\widetilde{A}^{(K)} = [a_{ij}^{(K)}]_{m\times n} \in M^{(K)}IFSM$, $\widetilde{B}^{(K)} = [b_{ij}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ and $\widetilde{C}^{(K)} = [c_{ij}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ then the following results hold.

(i) $\tilde{\phi}^{(K)} \subseteq \widetilde{A}^{(K)}$

(ii) $\widetilde{A}^{(K)} \subseteq \tilde{I}^{(K)}$

(iii) $\widetilde{A}^{(K)} \subseteq \widetilde{A}^{(K)}$

(iv) $\widetilde{A}^{(K)} \subseteq \widetilde{B}^{(K)}, \widetilde{B}^{(K)} \subseteq \widetilde{C}^{(K)} \Rightarrow \widetilde{A}^{(K)} \subseteq \widetilde{C}^{(K)}$ (Inclusion Law)

Proof

(i). For all i,j and K, $\mu_{\tilde{A}_{ij}}^{(K)} \geq 0$ and $\nu_{\tilde{A}_{ij}}^{(K)} \leq 1$, Hence $\tilde{\phi}^{(K)} \subseteq \widetilde{A}^{(K)}$.

(ii). For all i,j and K, $\mu_{\tilde{A}_{ij}}^{(K)} \leq 1$ and $\nu_{\tilde{A}_{ij}}^{(K)} \geq 0$, So $\widetilde{A}^{(K)} \subseteq \tilde{I}^{(K)}$

(iii). Proof is obvious.

(iv). For all i,j and K, $\mu_{\tilde{A}_{ij}}^{(K)} \leq \mu_{\tilde{B}_{ij}}^{(K)}$ and $\nu_{\tilde{A}_{ij}}^{(K)} \geq \nu_{\tilde{B}_{ij}}^{(K)}$,

Since $\widetilde{B}^{(K)} \subseteq \widetilde{C}^{(K)}$, $\mu_{\tilde{B}_{ij}}^{(K)} \leq \mu_{\tilde{C}_{ij}}^{(K)}$ and $\nu_{\tilde{B}_{ij}}^{(K)} \geq \nu_{\tilde{C}_{ij}}^{(K)}$, for all i,j and K.

Hence we have $\mu_{\tilde{A}_{ij}}^{(K)} \leq \mu_{\tilde{C}_{ij}}^{(K)}$ and $\nu_{\tilde{A}_{ij}}^{(K)} \geq \nu_{\tilde{C}_{ij}}^{(K)}$, for all i,j and K.

Hence $\widetilde{A}^{(K)} \subseteq \widetilde{C}^{(K)}$.

Definition 4.7

Let $\widetilde{A}^{(K)} = [a_{ij}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ then $\widetilde{A^{T}}^{(K)}$ is the Multi Intuitionistic Fuzzy Soft Transpose Matrix of $\widetilde{A}^{(K)}$ and is given by $\widetilde{A^{T}}^{(K)} = [a_{ji}^{(K)}]_{n\times m} \in M^{(K)}IVFSM$.

Proposition 4.8

Let $\widetilde{A}^{(K)} = [a_{ij}^{(K)}]_{m\times n} \in M^{(K)}IFSM$, $\widetilde{B}^{(K)} = [b_{ij}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ and $\widetilde{C}^{(K)} = [c_{ij}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ then

(i) $\widetilde{A}^{(K)} \subseteq \widetilde{B}^{(K)} \Rightarrow \widetilde{A}^{(K)} + \widetilde{B}^{(K)} = \widetilde{B}^{(K)}$

(ii) $\widetilde{A}^{(K)} + \widetilde{\phi}^{(K)} = \widetilde{A}^{(K)}$ (Identity Law)

(iii) $\widetilde{A}^{(K)} + \widetilde{I}^{(K)} = \widetilde{I}^{(K)}$ (Domination Law)

(iv) $\widetilde{A}^{(K)} + \widetilde{A}^{(K)} = \widetilde{A}^{(K)}$ (Idempotent Law)

(v) $\widetilde{A}^{(K)} + \widetilde{B}^{(K)} = \widetilde{B}^{(K)} + \widetilde{A}^{(K)}$ (Commutative Law)

(vi) $\left(\widetilde{A}^{(K)} + \widetilde{B}^{(K)}\right) + \widetilde{C}^{(K)} = \widetilde{A}^{(K)} + \left(\widetilde{B}^{(K)} + \widetilde{C}^{(K)}\right)$ (Associative Law)

(vii) $\left(\widetilde{A}^{(K)} + \widetilde{B}^{(K)}\right)^{T} = \widetilde{A^{T}}^{(K)} + \widetilde{B^{T}}^{(K)}$

(viii) $\left(\widetilde{A}^{(K)} + \widetilde{B}^{(K)} + \widetilde{C}^{(K)}\right)^{T} = \widetilde{A^{T}}^{(K)} + \widetilde{B^{T}}^{(K)} + \widetilde{C^{T}}^{(K)}$

(xi) $\left(\widetilde{A^{T}}^{(K)}\right)^{T} = \widetilde{A}^{(K)}$

Proof

Given for all i,j and K

$\widetilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} = \left[\left(\mu^{(K)}_{\widetilde{A}_{ij}}, \nu^{(K)}_{\widetilde{A}_{ij}}\right)\right] \in M^{(K)}IFSM$,

$\widetilde{B}^{(K)} = [b_{ij}{}^{(K)}]_{m\times n} = \left[\left(\mu^{(K)}_{\widetilde{B}_{ij}}, \nu^{(K)}_{\widetilde{B}_{ij}}\right)\right] \in M^{(K)}IFSM$, $\widetilde{\phi}^{(K)} = \left[\left(0^{(K)}, 1^{(K)}\right)\right] \in M^{(K)}IFSM$.

By the definition of Transpose, we have

$\widetilde{A^{T}}^{(K)} = [a_{ji}{}^{(K)}]_{n\times m} = \left[\left(\mu^{(K)}_{\widetilde{A}_{ji}}, \nu^{(K)}_{\widetilde{A}_{ji}}\right)\right] \in M^{(K)}IFSM$ and

$\widetilde{B^{T}}^{(K)} = [b_{ji}{}^{(K)}]_{n\times m} = \left[\left(\mu^{(K)}_{\widetilde{B}_{ji}}, \nu^{(K)}_{\widetilde{B}_{ji}}\right)\right] \in M^{(K)}IFSM$

(i) Since $\widetilde{A}^{(K)} \subseteq \widetilde{B}^{(K)}$, we have $\mu^{(K)}_{\widetilde{A}_{ij}} \le \mu^{(K)}_{\widetilde{B}_{ij}}$ and $\nu^{(K)}_{\widetilde{A}_{ij}} \ge \nu^{(K)}_{\widetilde{B}_{ij}}$.

$$\therefore\ \widetilde{A}^{(K)} + \widetilde{B}^{(K)} = \left[\left(\max\left(\mu^{(K)}_{\widetilde{A}_{ij}}, \mu^{(K)}_{\widetilde{B}_{ij}}\right), \min\left(\nu^{(K)}_{\widetilde{A}_{ij}}, \nu^{(K)}_{\widetilde{B}_{ij}}\right)\right)\right]$$

$$= \left[\left(\mu^{(K)}_{\widetilde{B}_{ij}}, \nu^{(K)}_{\widetilde{B}_{ij}}\right)\right]$$

$$= \widetilde{B}^{(K)}$$

(ii) For all i,j and K , $\widetilde{A}^{(K)} + \widetilde{\phi}^{(K)} = \left[\left(\max\left(\mu^{(K)}_{\widetilde{A}_{ij}}, 0^{(K)}\right), \min\left(\nu^{(K)}_{\widetilde{A}_{ij}}, 1\right)\right)\right]$

$$= \left[\left(\mu^{(K)}_{\widetilde{A}_{ij}}, \nu^{(K)}_{\widetilde{A}_{ij}}\right)\right]$$

$$= \widetilde{A}^{(K)}$$

(iii) Similar as (ii) by using the definition of $\tilde{I}^{(K)}$.

(iv) Proof is obvious.

(v) For all i,j and K, $\left(\widetilde{A}^{(K)} + \widetilde{B}^{(K)}\right) = \left[\left(\max\left(\mu_{\widetilde{A}_{ij}}^{(K)}, \mu_{\widetilde{B}_{ij}}^{(K)}\right), \min\left(\nu_{\widetilde{A}_{ij}}^{(K)}, \nu_{\widetilde{B}_{ij}}^{(K)}\right)\right)\right]$

$$= \left[\left(\max\left(\mu_{\widetilde{B}_{ij}}^{(K)}, \mu_{\widetilde{A}_{ij}}^{(K)}\right), \min\left(\nu_{\widetilde{B}_{ij}}^{(K)}, \nu_{\widetilde{A}_{ij}}^{(K)}\right)\right)\right]$$

$$= \widetilde{B}^{(K)} + \widetilde{A}^{(K)}$$

(vi) Similarly as (v)

(vii) For all i,j and K, $\left(\widetilde{A}^{(K)} + \widetilde{B}^{(K)}\right)^T = \left[\left(\max\left(\mu_{\widetilde{A}_{ij}}^{(K)}, \mu_{\widetilde{B}_{ij}}^{(K)}\right), \min\left(\nu_{\widetilde{A}_{ij}}^{(K)}, \nu_{\widetilde{B}_{ij}}^{(K)}\right)\right)\right]^T$

$$= \left[\left(\max\left(\mu_{\widetilde{A}_{ji}}^{(K)}, \nu_{\widetilde{A}_{ji}}^{(K)}\right), \min\left(\mu_{\widetilde{B}_{ji}}^{(K)}, \nu_{\widetilde{B}_{ji}}^{(K)}\right)\right)\right]$$

$$= \widetilde{A^{T}}^{(K)} + \widetilde{B^{T}}^{(K)}$$

(viii) Proof is similar to (vii)

(ix) $\left(\widetilde{A^{T}}^{(K)}\right)^T = \left[\left(\mu_{\widetilde{A}_{ji}}^{(K)}, \nu_{\widetilde{A}_{ji}}^{(K)}\right)\right]^T$

$$= \left[\left(\mu_{\widetilde{A}_{ij}}^{(K)}, \nu_{\widetilde{A}_{ij}}^{(K)}\right)\right]$$

$$= \widetilde{A}^{(K)}$$

Definition 4.9

Let $\widetilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ where $a_{ij}{}^{(K)} = (\mu_j^{(K)}(u_i), \nu_j^{(K)}(u_i))$. Then a Multi Intuitionistic Fuzzy Soft Square Matrix $\widetilde{A}^{(K)}$ is called a Multi Intuitionistic Fuzzy Soft Symmetric Matrix if $\widetilde{A}^{(K)} = \widetilde{A^{T}}^{(K)}$. i.e For all i,j and K. $[a_{ij}{}^{(K)}] = [a_{ji}{}^{(K)}]$.

Example 4.10

Let $\widetilde{A}_{2\times2}{}^{(2)} = \begin{pmatrix} ((0.8,0.2)(0.7,0.2)) & ((0.6,0.3)(0.5,0.4)) \\ ((0.6,0.3)(0.5,0.4)) & ((0.7,0.2)(0.6,0.2)) \end{pmatrix}_{2\times2}$ be a Multi Intuitionistic Fuzzy Soft Square Matrix. Then

$$\widetilde{A^{T}}^{(2)} = \begin{pmatrix} ((0.8,0.2)(0.7,0.2)) & ((0.6,0.3)(0.5,0.4)) \\ ((0.6,0.3)(0.5,0.4)) & ((0.7,0.2)(0.6,0.2)) \end{pmatrix}_{2\times2}$$

Then $\widetilde{A^{T}}^{(2)} = \widetilde{A}^{(2)}$. Hence $\widetilde{A}^{(2)}$ is Multi Intuitionistic Fuzzy Soft Symmetric Matrix.

Proposition 4.11

(i) Let $\widetilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ where $a_{ij}{}^{(K)} = (\mu_j^{(K)}(u_i),\nu_j^{(K)}(u_i))$ then

$\widetilde{A}^{(K)} + \widetilde{A^{T}}^{(K)}$ is Symmetric

(ii) If $\widetilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$, $\widetilde{B}^{(K)} = [b_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ are symmetric then

$\widetilde{A}^{(K)} + \widetilde{B}^{(K)}$ is Symmetric.

Proof

Let $\widetilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} = \left[\left(\mu_{\widetilde{A}_{ij}}^{(K)}, \nu_{\widetilde{A}_{ij}}^{(K)}\right)\right] \in M^{(K)}IFSM$,

Then $\widetilde{A^{T}}^{(K)} = [a_{ji}{}^{(K)}]_{n\times m} = \left[\left(\mu_{\widetilde{A}_{ji}}^{(K)}, \nu_{\widetilde{A}_{ji}}^{(K)}\right)\right] \in M^{(K)}IFSM$

Now $\left(\widetilde{A}^{(K)} + \widetilde{A^{T}}^{(K)}\right)^{T} = \widetilde{A^{T}}^{(K)} + \left(\widetilde{A^{T}}^{(K)}\right)^{T}$

$= \widetilde{A^{T}}^{(K)} + \widetilde{A}^{(K)}$ (by Proposition 4.8 (vii))

$= \widetilde{A}^{(K)} + \widetilde{A^{T}}^{(K)}$ (by Proposition 4.8 (xi))

Hence $\widetilde{A}^{(K)} + \widetilde{A^{T}}^{(K)}$ is Symmetric.

(ii). Since $\widetilde{A}_{n\times n}{}^{(K)}$ and $\widetilde{B}_{n\times n}{}^{(K)}$ are symmetric , for all i,j and K, $\mu_{\widetilde{A}_{ij}}^{(K)} = \mu_{\widetilde{A}_{ji}}^{(K)}$ and $\nu_{\widetilde{A}_{ij}}^{(K)} = \nu_{\widetilde{A}_{ji}}^{(K)}$.

$\left(\widetilde{A}^{(K)} + \widetilde{B}^{(K)}\right)^{T} = \widetilde{A^{T}}^{(K)} + \widetilde{B^{T}}^{(K)}$

$= \left[\left(\max(\mu_{\widetilde{A}_{ji}}^{(K)}, \mu_{\widetilde{B}_{ji}}^{(K)}), \min(\nu_{\widetilde{A}_{ji}}^{(K)}, \nu_{\widetilde{B}_{ji}}^{(K)})\right)\right]$

$= \left[\left(\max(\mu_{\widetilde{A}_{ij}}^{(K)}, \mu_{\widetilde{B}_{ij}}^{(K)}), \min(\nu_{\widetilde{A}_{ij}}^{(K)}, \nu_{\widetilde{B}_{ij}}^{(K)})\right)\right]$

$= \widetilde{A}^{(K)} + \widetilde{B}^{(K)}$

Hence $\widetilde{A}^{(K)} + \widetilde{B}^{(K)}$ is Symmetric

Definition 4.12

Let $\widetilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ where $a_{ij}{}^{(K)} = (\mu_j^{(K)}(u_i),\nu_j^{(K)}(u_i))$. Then $\widetilde{A^{C}}^{(K)}$, the Multi Intuitionistic Fuzzy Soft Complement Matrix $\widetilde{A}^{(K)}$ is defined as

$$\widetilde{A^{C}}^{(K)} = [b_{ij}{}^{(K)}]_{m\times n} = (\nu_j^{(K)}(u_i),\mu_j^{(K)}(u_i)) \text{ , For all i,j and K.}$$

Example 4.13

Let $\widetilde{A}_{2\times2}^{(2)} = \begin{pmatrix} ((0.7,0.3)(0.5,0.4)) & ((0.4,0.1)(0.6,0.2)) \\ ((0.4,0.5)(0.6,0.2)) & ((0.7,0.2)(0.8,0.1)) \end{pmatrix}_{2\times2}$

Then its Complement $\widetilde{A^C}^{(K)}$ is given by

$$\widetilde{A^C}_{2\times2}^{(2)} = \begin{pmatrix} ((0.3,0.7)(0.4,0.5)) & ((0.1,0.5)(0.2,0.6)) \\ ((0.5,0.4)(0.2,0.6)) & ((0.2,0.7)(0.1,0.8)) \end{pmatrix}_{2\times2}$$

Proposition 4.14

Let $\widetilde{A}^{(K)} = [a_{ij}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ and $\widetilde{B}^{(K)} = [b_{ij}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ then

(i) $\left(\widetilde{A^C}^{(K)}\right)^C = \widetilde{A}^{(K)}$ (Double Complements (or) Involution Law)

(ii) $\widetilde{\phi^C}^{(K)} = \tilde{I}^{(K)}$, $\widetilde{I^C}^{(K)} = \tilde{\phi}^{(K)}$ (Complements Law)

(iii) $\left(\widetilde{A}^{(K)} + \tilde{I}^{(K)}\right)^C = \tilde{\phi}^{(K)}$

(iv) $\left(\widetilde{A}^{(K)} + \widetilde{B}^{(K)}\right)^C = \left(\widetilde{B}^{(K)} + \widetilde{A}^{(K)}\right)^C$

(v) $\left(\widetilde{A}^{(K)} + \widetilde{B}^{(K)} + \widetilde{C}^{(K)}\right)^C = \left(\widetilde{C}^{(K)} + \widetilde{B}^{(K)} + \widetilde{A}^{(K)}\right)^C$

(vi) $\left(\widetilde{A^C}^{(K)}\right)^T = \left(\widetilde{A^T}^{(K)}\right)^C$

(vii) $\left(\widetilde{A^C}^{(K)} + \widetilde{B^C}^{(K)}\right)^T = \left(\widetilde{A^C}^{(K)}\right)^T + \left(\widetilde{B^C}^{(K)}\right)^T$

Proof

Given $\widetilde{A}^{(K)} = [a_{ij}^{(K)}]_{m\times n} = \left[\left(\mu_{\tilde{A}_{ij}}^{(K)}, \nu_{\tilde{A}_{ij}}^{(K)}\right)\right] \in M^{(K)}IFSM$ and

$\widetilde{B}^{(K)} = [b_{ij}^{(K)}]_{m\times n} = \left[\left(\mu_{\tilde{B}_{ij}}^{(K)}, \nu_{\tilde{B}_{ij}}^{(K)}\right)\right] \in M^{(K)}IFSM$ for all i,j and K.

(i). Consider $\left(\widetilde{A^C}^{(K)}\right)^C = \left[\left(\nu_{\tilde{A}_{ij}}^{(K)}, \mu_{\tilde{A}_{ij}}^{(K)}\right)\right]$

$= \left[\left(\mu_{\tilde{A}_{ij}}^{(K)}, \nu_{\tilde{A}_{ij}}^{(K)}\right)\right]$

$= \widetilde{A}^{(K)}$

(ii)Proof follows from the Definition 3.4 and Definition 3.5 and Definition 4.12.

(iii)$\left(\tilde{A}^{(K)}+\tilde{I}^{(K)}\right)^{C} = \tilde{I}^{c\,(K)} = \tilde{\phi}^{(K)}$ (by using Proposition 4.8 (iii) and For all i,j and K)

(iv). Consider $\left(\tilde{A}^{(K)}+\tilde{B}^{(K)}\right)^{C} = \left[\left(\max\left(\mu_{\tilde{A}_{ij}}^{(K)}, \mu_{\tilde{B}_{ij}}^{(K)}\right), \min\left(\nu_{\tilde{A}_{ij}}^{(K)}, \nu_{\tilde{B}_{ij}}^{(K)}\right)\right)\right]^{C}$

$$= \left[\left(\min\left(\nu_{\tilde{A}_{ij}}^{(K)}, \nu_{\tilde{B}_{ij}}^{(K)}\right), \max\left(\mu_{\tilde{A}_{ij}}^{(K)}, \mu_{\tilde{B}_{ij}}^{(K)}\right)\right)\right]$$

$$= \left[\left(\min\left(\nu_{\tilde{B}_{ij}}^{(K)}, \nu_{\tilde{A}_{ij}}^{(K)}\right), \max\left(\mu_{\tilde{B}_{ij}}^{(K)}, \mu_{\tilde{A}_{ij}}^{(K)}\right)\right)\right]$$

$$= \left[\left(\max\left(\mu_{\tilde{B}_{ij}}^{(K)}, \mu_{\tilde{B}_{ij}}^{(K)}\right), \min\left(\nu_{\tilde{B}_{ij}}^{(K)}, \nu_{\tilde{A}_{ij}}^{(K)}\right)\right)\right]$$

$$= \left(\tilde{B}^{(K)}+\tilde{A}^{(K)}\right)^{C}$$

(v) Proof follows similar to (iv).

(vi) For all i,j and K, $\left(\widetilde{A^{C}}^{(K)}\right)^{T} = \left[\left(\nu_{\tilde{A}_{ij}}^{(K)}, \mu_{\tilde{A}_{ij}}^{(K)}\right)\right]^{T}$

$$= \left[\left(\nu_{\tilde{A}_{ji}}^{(K)}, \mu_{\tilde{A}_{ji}}^{(K)}\right)\right] \text{-----}\rightarrow(1)$$

Now, $\left(\widetilde{A^{T}}^{(K)}\right)^{C} = \left[\left(\mu_{\tilde{A}_{ji}}^{(K)}, \nu_{\tilde{A}_{ji}}^{(K)}\right)\right]^{C}$

$$= \left[\left(\nu_{\tilde{A}_{ji}}^{(K)}, \mu_{\tilde{A}_{ji}}^{(K)}\right)\right] \text{----------------------}\rightarrow(2)$$

From (1) and (2), $\left(\widetilde{A^{C}}^{(K)}\right)^{T} = \left(\widetilde{A^{T}}^{(K)}\right)^{C}$

(vii)Proof follows from the Proposition 4.8 (vii)

Remark 4.15

If $\tilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$, $\tilde{B}^{(K)} = [b_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ and $\tilde{C}^{(K)} = [c_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ then,

(i) $\left(\tilde{A}^{(K)}+\tilde{B}^{(K)}\right)^{C} \neq \widetilde{A^{C}}^{(K)}+\widetilde{B^{C}}^{(K)}$

(ii) $\left(\tilde{A}^{(K)}+\tilde{B}^{(K)}+\tilde{C}^{(K)}\right)^{C} \neq \widetilde{A^{C}}^{(K)}+\widetilde{B^{C}}^{(K)}+\widetilde{C^{C}}^{(K)}$

Definition 4.16

Let $\widetilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}$IFSM where $a_{ij}{}^{(K)} = (\mu_j^{(K)}(u_i),\nu_j^{(K)}(u_i))$. Then Scalar Multiple of Multi Intuitionistic Fuzzy Soft Matrix $\widetilde{A}^{(K)}$ by a scalar C is defined as

$$C\,\widetilde{A}^{(K)} = [C\, a_{ij}{}^{(K)}]_{m\times n}, \text{ where } 0 \le C \le 1.$$

Example 4.17

Let $\widetilde{A}_{2\times1}{}^{(2)} = \begin{pmatrix} ((0.8,0.1)(0.4,0.5)) \\ ((0.7,0.3)(0.4,0.6)) \end{pmatrix}_{2\times1}$ be Multi Intuitionistic Fuzzy Soft Matrix then the Scalar Multiple of this matrix by a Scalar C = 0.5 is given by

$$C\,\widetilde{A}_{2\times1}{}^{(2)} = \begin{pmatrix} ((0.4,0.05)(0.2,0.25)) \\ ((0.35,0.15)(0.2,0.3)) \end{pmatrix}_{2\times1}$$

Proposition 4.18

Let $\widetilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}$IFSM where $a_{ij}{}^{(K)} = (\mu_j^{(K)}(u_i),\nu_j^{(K)}(u_i))$. If m,n are two Scalars such that $0 \le m,n \le 1$ then

(i) $\widetilde{A}^{(K)} \subseteq \widetilde{B}^{(K)} \Rightarrow m\widetilde{A}^{(K)} \subseteq n\widetilde{B}^{(K)}$

(ii) $\left(m\widetilde{A}^{(K)}\right)^T = m\widetilde{A^T}^{(K)}$

(iii) $\left(m\widetilde{A^C}^{(K)}\right)^T = m\left(\widetilde{A^C}^{(K)}\right)^T$

(iv) $m\left(n\widetilde{A}^{(K)}\right) = (mn)\,\widetilde{A}^{(K)}$

(v) $m\left(n\widetilde{A^T}^{(K)}\right) = (mn)\,\widetilde{A^T}^{(K)}$

(vi) $m\left(n\widetilde{A^C}^{(K)}\right) = (mn)\,\widetilde{A^C}^{(K)}$

Proof

(i)Since $\widetilde{A}^{(K)} \subseteq \widetilde{B}^{(K)}$, we have $\mu_{\widetilde{A}_{ij}}^{(K)} \leq \mu_{\widetilde{B}_{ij}}^{(K)}$ and $\nu_{\widetilde{A}_{ij}}^{(K)} \geq \nu_{\widetilde{B}_{ij}}^{(K)}$ for all i,j and K.

Now, $m(\widetilde{A}^{(K)}) = \left[\left(m\,\mu_{\widetilde{A}_{ij}}^{(K)}, m\,\nu_{\widetilde{A}_{ij}}^{(K)}\right)\right]$

$$= m\left[\left(\mu_{\widetilde{A}_{ij}}^{(K)}, \nu_{\widetilde{A}_{ij}}^{(K)}\right)\right]$$

$$\leq m\left[\left(\mu_{\widetilde{B}_{ij}}^{(K)}, \nu_{\widetilde{B}_{ij}}^{(K)}\right)\right]$$

$$\leq m\,(\widetilde{B}^{(K)})$$

(ii)Obvious from the Definition 4.7 and Definition 4.16.

(iii)For all i,j and K, $\left(m\widetilde{A^{C}}^{(K)}\right)^{T} = m\left[\left(\nu_{\widetilde{A}_{ij}}^{(K)},\ \mu_{\widetilde{A}_{ij}}^{(K)}\right)\right]^{T}$

$$= m\left[\left(\nu_{\widetilde{A}_{ji}}^{(K)},\ \mu_{\widetilde{A}_{ji}}^{(K)}\right)\right]$$

$$= m\left[\left(\nu_{\widetilde{A}_{ji}}^{(K)},\ \mu_{\widetilde{A}_{ji}}^{(K)}\right)\right]^{C}$$

$$= m\left(\widetilde{A^{T}}^{(K)}\right)^{C}$$

(iv) For all i,j and K, $m\left(n\widetilde{A}^{(K)}\right) = m\left[\left(n\,\mu_{\widetilde{A}_{ij}}^{(K)},\ n\,\nu_{\widetilde{A}_{ij}}^{(K)}\right)\right]$

$$= mn\left[\left(\mu_{\widetilde{A}_{ij}}^{(K)}, \nu_{\widetilde{A}_{ij}}^{(K)}\right)\right]$$

$$= (mn)\,\widetilde{A}^{(K)}$$

(v) For all i,j and K, $m\left(n\widetilde{A^{T}}^{(K)}\right) = m\left[\left(n\,\mu_{\widetilde{A}_{ij}}^{(K)},\ n\,\nu_{\widetilde{A}_{ij}}^{(K)}\right)\right]^{T}$

$$= mn\left[\left(\mu_{\widetilde{A}_{ij}}^{(K)},\ \nu_{\widetilde{A}_{ij}}^{(K)}\right)\right]^{T}$$

$$= (mn)\,\widetilde{A^{T}}^{(K)}$$

(vi) For all i,j and K, $m\left(n\widetilde{A^{C}}^{(K)}\right) = m\left[\left(n\,\mu_{\widetilde{A}_{ij}}^{(K)},\ n\,\nu_{\widetilde{A}_{ij}}^{(K)}\right)\right]^{C}$

$$= m\left[\left(n\,\nu_{\widetilde{A}_{ij}}^{(K)},\ n\,\mu_{\widetilde{A}_{ij}}^{(K)}\right)\right]$$

$$= mn \left[\left(\nu_{\tilde{A}_{ij}}^{(K)}, \ \mu_{\tilde{A}_{ij}}^{(K)} \right)\right]$$

$$= (mn)\ \widetilde{A^{c}}^{(K)}$$

Definition 4.19

Let $\tilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{m\times n} \in M^{(K)}IFSM$ where $a_{ij}{}^{(K)} = (\mu_j^{(K)}(u_i), \nu_j^{(K)}(u_i))$. Then Trace of Multi Intuitionistic Fuzzy Soft Matrix $\tilde{A}^{(K)}$ is

$$tr\ (\tilde{A}^{(K)}) = \left(\max\left(\mu_j^{(K)}(u_i)\right), \min\left(\nu_j^{(K)}(u_i)\right)\right)$$

Example 4.20

$$\text{Let } \tilde{A}_{2\times 2}{}^{(2)} = \begin{pmatrix} ((0.6,0.2)(0.7,0.2)) & ((0.6,0.3)(0.8,0.1)) \\ ((0.5,0.4)(0.8,0.1)) & ((0.7,0.2)(0.8,0.2)) \end{pmatrix}$$

Be a Multi Intuitionistic Fuzzy Soft Matrix , then trace of this matrix is

$$tr\ (\tilde{A}^{(K)}) = ((0.7,0.2)(0.8,0.2))$$

Proposition 4.21

Let $\tilde{A}^{(K)} = [a_{ij}{}^{(K)}]_{n\times n} \in M^{(K)}IFSM$ where $a_{ij}{}^{(K)} = (\mu_j^{(K)}(u_i), \nu_j^{(K)}(u_i))$. If m is a Scalar such that $0 \le m \le 1$ then

(i) $tr\ (\tilde{A}^{(K)}) = tr\ (\widetilde{A^{T}}^{(K)})$

(ii) $tr\ (m\ \tilde{A}^{(K)}) = tr\ (m\ \widetilde{A^{T}}^{(K)})$

(iii) $tr\ \left(\tilde{A}^{(K)} + \tilde{B}^{(K)}\right)^{T} = tr\ \left(\widetilde{A^{T}}^{(K)} + \widetilde{B^{T}}^{(K)}\right)$

(iv) $tr\ (m\ \tilde{A}^{(K)}) = (m)\ (\ tr\ (\ \tilde{A}^{(K)}))$

(v) $tr\ (m\ \widetilde{A^{T}}^{(K)}) = (m)\ (\ tr\ (\ \widetilde{A^{T}}^{(K)}))$

(vi) $tr\ (m\ \widetilde{A^{c}}^{(K)}) = (m)\ (\ tr\ (\ \widetilde{A^{c}}^{(K)}))$

Proof

(i)For all j and K, $tr\ (\tilde{A}^{(K)}) = \left(\max\left(\mu_j^{(K)}(u_j)\right), \min\left(\nu_j^{(K)}(u_j)\right)\right)$

$$= tr\ (\widetilde{A^{T}}^{(K)})$$

(ii)For all j and K, $tr\ (m\ \tilde{A}^{(K)}) = \left(\max\left(m\ \mu_j^{(K)}(u_j)\right), \min\left(m\ \nu_j^{(K)}(u_j)\right)\right)$

$= \text{tr}(m\, \widetilde{A^T}^{(K)})$

(iii)Proof follows by using the Proposition 4.8 (vii) and above result (i)

(iv)For all j and K, $\text{tr}(m\, \widetilde{A}^{(K)}) = \left(\max\left(m\, \mu_j^{(K)}(u_j)\right), \min\left(m\, \nu_j^{(K)}(u_j)\right)\right)$

$= m\left(\max\left(\mu_j^{(K)}(u_j)\right), \min\left(\nu_j^{(K)}(u_j)\right)\right)$

$= (m)\,(\text{tr}(\widetilde{A}^{(K)}))$

(v) For all j and K, $\text{tr}(m\, \widetilde{A^T}^{(K)}) = \left(\max\left(m\, \mu_j^{(K)}(u_j)\right), \min\left(m\, \nu_j^{(K)}(u_j)\right)\right)$

$= m\left(\max\left(\mu_j^{(K)}(u_j)\right), \min\left(\nu_j^{(K)}(u_j)\right)\right)$

$= (m)\ \text{tr}(\widetilde{A^T}^{(K)})$

(vi)For all j and K, $\text{tr}(m\, \widetilde{A^C}^{(K)}) = \left(\max\left(m\, \mu_j^{(K)}(u_j)\right), \min\left(m\, \nu_j^{(K)}(u_j)\right)\right)$

$= m\left(\max\left(\nu_j^{(K)}(u_j)\right), \min\left(\mu_j^{(K)}(u_j)\right)\right)$

$= (m)\ \text{tr}(\widetilde{A^C}^{(K)})$

Conclusion

In this paper we have defined a new matrix namely Multi intuinistic fuzzy soft matrix and studied its properties. We have defined three operation namely Transpose , Complement and Trace on a newly defined matrix. Also we have studied various properties of this matrix based on the operators defined. The concepts are illustrated with suitable examples.

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